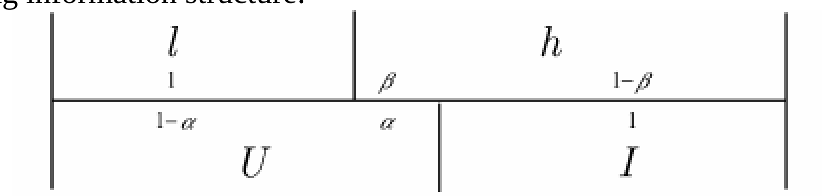
Feinberg and Skrzypacz (2005)

One-sided repeated offers game: Seller makes offers at time points 0, Δ, 2Δ, …, which the buyer can accept or reject. Seller's valuation is 0.

Benchmark case (common knowledge of first-order beliefs): Buyer's valuation is distributed according to CDF $Q : [l, h] \to [0, 1]$ with $l > 0$ and $Q(l + \varepsilon) > 0$ for all $\varepsilon > 0$. Buyer knows his valuation, but the seller has belief Q about the buyer's valuation. First-order beliefs are common knowledge. For this case, the equilibrium has a "Coase property:" as offers become increasingly frequent, actual time to reaching agreement goes to 0 and the price at which agreement is reached goes to l.

Now, with an example that has uncertainty about 1st order beliefs, let us show that there may be a delay even as offers become more and more frequent. We are concerned with the following information structure:



The buyer has a valuation of l or h. There are two types of seller:

- I, informed type, who knows that the buyer has valuation h
- U, uninformed type, who believes that the buyer has valuation h w/ probability α

The buyer with valuation h believes that the seller is type U with probability β.

The main result of the paper is that there are constants $\varepsilon, \gamma > 0$ such that $\text{Prob}(\tau > \varepsilon) > \gamma$ for all Δ, where τ is the stopping time when agreement is reached in equilibrium.

<u>Intuition why there is delay</u>: If there was no delay, then type I would be able to convince buyer h and sell at price h fairly quickly (an argument using the intuitive criterion and condition R – see below – is needed to reach this conclusion). By imitating type I, type U would be able to sell at price h to type h fairly quickly, and then sell at l to type l. The buyer of type l gets the price of l. The buyer of type h can imitate type l and get a price of l from U fairly quickly. By adding up, one can verify that the expected payoffs of each type add to more than the total surplus.

<u>Construction of an equilibrium with delay.</u>

We will convey the equilibrium texture by constructing an equilibrium in continuous time. There are many equilibria. Let us look for an equilibrium in which

- Types become fully revealed by time $T > 0$
- Seller type I raises price $p(t)$ slowly and reaches level h at time T
- Seller U imitates the path $p(t)$ until he "concedes" by reducing the price to l. Denote the equilibrium CDF of concession times by F, and let us look for an equilibrium in which F is strictly increasing and $F(T) = 1$.

- Buyer l buys when the price drops to l, but not before.
- Buyer h always immediately buys at price l, but mixes between buying or not at price p(t). Denote the CDF of purchases of buyer h at price p(t) by G, and let us look for an equilibrium in which G is strictly increasing and G(T) = 1.

The equilibrium is characterized by three functions: p, F and G, and two equilibrium conditions: the indifference of types U and h (both of these types are mixing). Because there are two conditions for three functions, there are many solutions. Let us look for an equilibrium with price path

$$p_t = h\, e^{-r(T-t)}.$$

This price path is chosen, quite arbitrarily, to simplify the equilibrium conditions below. The payoff to the buyer of type h if he "concedes" at time t is

$$\int_0^t e^{-rs}(h-l)\beta f(s)ds + e^{-rt}(h-p(t))(1-\beta F(t))$$

This expression has to be constant in t. Differentiating with respect to t, we find that

$$(h-l)\beta f(t) - r(h-p(t))(1-\beta F(t)) - p'(t)(1-\beta F(t)) - (h-p(t))\beta f(t) = 0$$

Using $p'(t) = rp(t)$, we find that

$$(p(t)-l)\beta f(t) = rh(1-\beta F(t)) \qquad (*)$$

Similarly, we write the payoff of the seller of type U as

$$\int_0^t e^{-rs} p(s)\alpha g(s)ds + e^{-rt} l(1-\alpha G(t))$$

Differentiating with respect to t, we find that

$$(p(t)-l)\alpha g(t) = rl(1-\alpha G(t)) \qquad (**)$$

To solve equations (*) and (**), note that $\log(1-\alpha G(t))' = -\frac{\alpha g(t)}{1-\alpha G(t)}$, so equations (*) and (**) become

$$\log(1-\beta F(t))' = -\frac{rh}{p(t)-l} \quad \text{and} \quad \log(1-\alpha G(t))' = -\frac{rl}{p(t)-l},$$

so

$$\log(1-\beta F(t)) = -\int_0^t \frac{rh}{p(s)-l}ds - K_F \quad \text{and} \quad \log(1-\alpha G(t)) = -\int_0^t \frac{rl}{p(s)-l}ds - K_G$$

We could have $K_F > 0$ or $K_G > 0$, but not both. The case $K_F = K_G = 0$ occurs when

$$\log(1-\beta)/h = \log(1-\alpha)/l \qquad (***)$$

Exercise: Find K_F and K_G for the case when (***) fails.

Proving that Delay Must Occur.

The paper proves that delay must occur in any equilibrium that satisfies the intuitive criterion and condition R:

We say that the equilibrium satisfies ***condition R*** *(for revelation) if once there is common belief that a seller's type was revealed, then the outcome of the game is the same as if this was the only type of seller.*

In other words, revelation leads to a reduced game: once the seller's type is revealed, the equilibrium outcome follows an equilibrium outcome of the game with the reduced information structure.

Theorem. *In an equilibrium that satisfies the intuitive criterion and condition R, delay in bargaining occurs with a strictly positive probability, i.e. even when offers are made frequently the real time until agreement occurs is bounded away from 0 with positive probability.*

Sketch of proof. Suppose agreement is reached "soon" in a game with frequent offers. Denote by π^I and π^U the expected equilibrium payoffs of the informed and the uninformed types. By imitating the informed type, type U can get a payoff of π^I from a buyer of type h and reach an agreement with h fairly soon. Thereafter, type U can offer l and sell to buyer l (after a slight delay). Therefore,

$$\pi^U \geq \alpha\pi^I + (1-\alpha)l$$

is nearly satisfied. Let $\bar{t}$ be defined by

$$\delta^{\bar{t}}(\alpha h + (1-\alpha)l) = \alpha\pi^I + (1-\alpha)l \qquad (1)$$

Then seller U could never benefit from delaying agreement until slightly after time $\bar{t}$ by asking price 2h. If we had $\delta^{\bar{t}} h > \pi^I$, then by the intuitive criterion, the buyer must assign probability 1 to type I if he observes such delay, implying that I has a profitable deviation. This is impossible, so we conclude that

$$\delta^{\bar{t}} h \leq \pi^I \qquad (2)$$

is nearly satisfied. (1) and (2) imply that $\pi^I \geq h$ nearly holds, i.e. type I gets a payoff close to *h*.

Then type U gets profit close to $\alpha h + (1-\alpha)l$, so he sells at price h to type h and at price l to type l fairly quickly.

Now, buyer h by imitating l can get price l from seller U, so his payoff is at least $\beta(h - l)$. This leads to a contradiction: if we add up players' expected payoff, we get more than total surplus. QED

Remark: If there was no perfectly informed type of seller, delay would not occur.