Economics 209A
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Due Friday, December 9, 2005

Problem Set 6.

Directions: Make every attempt to do each problem on your own. If you need help, please feel free to consult papers or discuss the problems with your classmates. On your solutions, please acknowledge any references you used or help you got, e.g. "I discussed this problem with so and so." In your formal proofs, please be concise.

Problem 1.

Consider education signaling game with three types, whose natural productivities are $x_L < x_M < x_H$. Assume that education is unproductive and per-unit cost of getting education is $c_L > c_M > c_H$ for each of the three types. Market pays perceived productivity to each worker type.

(a) Please characterize all triples of education levels (e_L, e_M, e_H) that are part of a separating equilibrium.

(b) Which of those triples are part of a separating equilibrium that satisfies the intuitive criterion? For each triple, specify off-equilibrium path beliefs that satisfy the intuitive criterion.

(c) Of the equilibria in part (b), prove that the only one that passes the D1 criterion is the least cost separating equilibrium.

Problem 2.

Consider the Prisoners' Dilemma game from Sannikov (2005). Recall that players learn about each other's actions through signals

$$dX_t^1 = A_t^1 \, dt + dZ_t^1 \qquad dX_t^2 = A_t^2 \, dt + dZ_t^2$$

and expected stage-game payoffs are given by $g_i(a_i, a_j) = 2a_j - a_i$. Prove that $\mathcal{E}(r)=\mathcal{N}$ for $r = 3$.

Hint: Derive an upper bound on the curvature of $\mathcal{E}(r)$ from the optimality equation, and show the curvature of any convex smooth subset of V^* must exceed this upper bound.

Bonus: Using paper-and-pencil calculations, find as small value of r as you can, for which $\mathcal{E}(r)=\mathcal{N}$.

Problem 3.

This problem intends to apply the ideas of Fudenberg, Levine and Maskin (1994) to PPE in pure strategies. Assume for simplicity that there are only two players. First, let us recall the following two definitions from FLM.

Definition 1. A pure-action profile *a* is *enforceable* if there exists a map $v : Y \to R^n$ such that

$$g_i(a) + \sum_{y \in Y} \pi(y \mid a) v_i(y) \geq g_i(a_i', a_{-i}) + \sum_{y \in Y} \pi(y \mid a_i', a_{-i}) v_i(y)$$

for all $i = 1,2$ and $a_i' \neq a_i$.

Definition 2. A pure-action profile *a* is *pairwise identifiable* if the rank of matrix

$$\Pi_{12}(a) = \begin{pmatrix} \Pi_1(a_2) \\ \Pi_2(a_1) \end{pmatrix}$$

equals rank $(\Pi_1(a_2))$ + rank $(\Pi_2(a_1))$ - 1.

Also, recall Lemma 5.5 from FLM:

Lemma. If a pure-action profile is enforceable and pairwise-identifiable, then it is enforceable with respect to all regular hyperplanes.

Now, assuming that all pure-strategy profiles in a certain game are enforceable and pairwise-identifiable, sketch a proof the pure minmax-threat Folk Theorem for PPE in pure strategies.