Economics 209A
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Hints to Problem Set 4 (due **Wednesday, November 2**)

Problem 1.

It is argued in mergers and acquisitions that a raider may benefit from acquiring a toehold before making a takeover bid. This problem is designed to investigate this claim.

There are two bidders in a potential takeover with valuations for the target v_1 and v_2 distributed independently uniformly on [0, 1]. Suppose that the target's value is 0 in case a take-over fails. Bidders 1 and 2 own fractions α_1 and $\alpha_2 \in [0, ½)$ of the target's shares before the auction. Assume that bidder 1 has a bigger toehold, i.e. $\alpha_1 > \alpha_2$. The target owns the remaining fraction $1 - \alpha_1 - \alpha_2 \in [½, 1]$ of its shares. Each bidder needs to acquire ½ of the target's shares to gain control over the target. If bidder *i* gains control over the target, the target's value rises to v_i.

Suppose that the target designs an optimal auction to determine its acquirer. Assume that the target can commit to such an auction. If bidder *i* wins the auction, the target needs to give it $½ - \alpha_i$ of its shares.

(a) If bidder 1 wins the auction and pays x to the acquirer, what is his payoff?

(b) If bidder 2 wins the auction and bidder 1 pays x to the acquirer, what is bidder 1's payoff?

When answering this question, do not forget that the value of the firm increases when bidder 2 takes it over, so bidder 1's shares are worth something.

(c) Following Myerson (1981), please design an optimal auction and derive bidder 1's and bidder 2's expected utilities as functions of their valuations. Which bidder gets a higher payoff? Interpret.

Remark. Assume that if one of the bidders refuses to participate in the auction, then the target does not hold auction altogether. For example, if bidder 1 refuses to participate, then bidder 2 does not acquire the target and the toehold of bidder 1 has value 0. Also, assume that the target knows the toeholds of each bidder because by law, potential acquirers are required to report their toeholds.

Do not let parts (a) and (b) confuse you. It is best to start solving part (c) from scratch, forgetting what you did in parts (a) and (b). Denote by $p_i(v_1, v_2)$ the probability that bidder i wins the auction if the announcements are v_1 and v_2. As in Myerson (1981), write the expression for total surplus. Then find $\frac{dU_i(p,x,v_i)}{dv_i}$, *which will be a bit different from Myerson because bidder i's valuation of 50% of target's shares is not* v_i

but $v_i/2$. Write the integral for bidder i's expected utility, and write the seller's share of total surplus. Maximize. Plot the allocation determined by the optimal auction. Find if the bidder with a greater toehold has any advantage. Although it is not required for this problem, try to think of a natural way to implement this auction. If interested, see Betton, Eckbo and Thorburn "The Zero-Toehold Puzzle," working paper, Dartmouth College. This paper does use an optimal-auction approach, but provides empirical evidence that acquirers tend not to get toeholds in takeovers, and suggests some theoretical explanations. The paper argues that the target will resist acquirers who choose to obtain a toehold, and cooperate with acquirers who do not.

Problem 2.

Consider a Rubinstein bargaining setting, in which both players have the same constant cost *c* of waiting per period.

(a) Find A and B.

Draw d_1 and d_2 and find their intersection.

(b) Find the range of values of *c*, for which there exists a SPE in which agreement is not reached in the first period. Construct such a SPE. As building blocks, you may use SPE with immediate agreement and any value in Δ for the player who makes an offer without elaborating on what happens in those SPE.

You need to prove that for values of c outside the range agreement must be reached in the first period. For such a proof, try to argue by contradiction. Assume that agreement is not reached in period 1 and using part (a), find a lower bound on 2's payoff in the second period. From this, find an upper bound on 1's payoff in the second period, and find how much that upper bound is worth to player 1 in period 1. Show that this is less than any payoff from A, and derive a contradiction.

You also need to construct a SPE with disagreement in period 1 for values c within the possible range. You will need to specify player 1's first-period offer (which is rejected). Also, for any offer of player 1 in period 1, specify player 2's decision to accept or reject and the SPE that follows in period 2 if player 2 rejects in period 1. Use an SPE with immediate agreement for each subgame in period 2.

Problem 3.

Consider the following general set of mechanisms (M, *a*) in the costly state verification framework. The entrepreneur has to choose some message from a message space M, which determines action according to a rule $a : M \to A$. Actions can consist of a transfer payment (where the entrepreneur can only choose messages that yield feasible transfer payments), and a deterministic decision rule by the lender on whether to verify. If the lender does verify, the action can include a feasible payment that depends on the type verified. In this setting with perfect commitment, we can invoke the revelation principle (as we did in class) and characterize an optimal mechanism. There are, of course, many other mechanisms (that need not be truth-telling direct revelation mechanisms) that implement the same outcome as our standard debt contract. Let this set of optimal mechanisms be called O.

Now suppose that the lender cannot commit through a mechanism to verify; but rather, the mechanism only gives the lender the right to verify, which the lender may choose not to exercise. The lender will only exercise this right when it is in her interest to do so, given equilibrium actions. We call a mechanism "credible" if the lender chooses to verify whenever he has the right to do so. Whether or not a mechanism in O is credible will of course depend on the exogenously given prior distribution F of cash outcomes, the cost of verification, and D (which depends on the exogenous variable K).

A. Identify the "most credible" mechanism in O. In particular, characterize a mechanism in O that is credible for any set of exogenous parameters (F, c, D) which admits any credible mechanism in O.

The most credible mechanism involves two messages.

B. Under what conditions on (F, c, D) will this "most credible" mechanism in fact be credible?

This mechanism will be credible if the expected amount of funds that the lender can extract from verifying (when he has the right to do so) is bigger or equal than the cost of monitoring.

<u>Problem 4.</u>

Consider the war of attrition from the first lecture and from problem 1 on problem set 1, except that now both players may be behavioral with probabilities p_1 for player 1 and p_2 for player 2. Recall that two players initially make demands (2,1) and (1,2) about how to split $3, and wait until one of the players concedes. Both players discount future payoffs at the common rate r. The behavioral type is not able to concede. The purpose of this problem is to characterize all mixed strategy Bayesian Nash equilibria. Denote by F_1 the CDF of concession times of the *normal* type of player 1, and by F_2 the CDF of concession times of the *normal* type of player 2.

A. Sketch a proof for each of the following claims. Be as concise as possible.

(1) F_1 and F_2 reach 1 at the same time at some time $T \geq 0$.
(2) F_1 and F_2 have no atoms, except possibly at 0.
(3) F_1 and F_2 are strictly increasing on [0, T].

Although this problem is hard, you should be able to do it by following the solution to problem 1 in problem set 1.

B. Find all Bayesian Nash equilibria, and justify your logic.

You will have two differential equations that define the equilibrium, and you need to find the right boundary conditions in order to solve them. The boundary conditions come from the facts that (1) F_1 and F_2 reach 1 at the same time and (2) F_1 and F_2 cannot both have an atom at 0, although one of the two functions may (and must) have an atom at 0.

Problem 5.

Consider the following version of the education signaling model. There is a continuum of types of workers with skill levels $t \in [0, 2]$. Skill level is unobservable, but employers can see worker's education e, and infer from it the worker's type. The cost of getting education e to type t is $(t - e)^2$, where t is the "bliss" education level. If the market believes that a worker is of type t, it will pay the worker wage 2t. Find the fully separating signaling equilibrium.

The Leland and Pyle exercise from class should be a good hint how to derive the ODE for the fully separating equilibrium. The appropriate boundary condition is that the "worst" type takes his "bliss" education level. You can find a closed form expression for the market inference as a function of education level, but you may not be able to invert it in closed form.

Problem 6.

Recall the setting of Morris and Shin (1998). The state of fundamentals θ is uniformly distributed on the interval [0, 1]. Suppose that the currency is initially pegged at $e^* = 2$, and the exchange rate in the absence of government intervention is given by $f(\theta) = ½+\theta$. There is a unit mass of speculators, each of whom gets a signal x about the state of fundamentals θ uniformly distributed on $[\theta - \varepsilon, \theta + \varepsilon]$ where ε is relatively small. The signals are independent across speculators, conditional on θ. After receiving their signals, the speculators simultaneously decide whether to attack the currency or not. A speculator's payoff is $e^* - f(\theta) - t$ he attacks and the currency is devalued, and $-t$ if attacks and the currency is defended, where $t = 1$ is the transaction cost. If a speculator does not attack, he gets a payoff of 0.

After seeing the mass of speculators who attack, the government decides whether to defend the exchange rate or not. The government derives value $v = 1$ from defending the exchange rate, but has to pay a cost of $c(\alpha, \theta) = 1.2 + \alpha - \theta$, where α is the mass of speculators who attack.

The objective of this problem is to characterize the unique equilibrium.

(a) Find the state of fundamentals $\underline{\theta}$, such that for $\theta < \underline{\theta}$ the government will not defend the currency even if nobody attacks.

(b) Find the state of fundamentals $\bar{\theta}$, such that for $\theta > \bar{\theta}$ a speculator would get a negative payoff from attacking even if the currency is devalued for sure.

(c) Find a(θ), the critical mass of speculators that triggers a devaluation when the state of fundamentals is θ.

(d) Conjecture that a speculator attacks if and only if his signal $x \leq x^*$. Assuming $x^* \in [\underline{\theta} + \varepsilon, 1-\varepsilon]$, plot the mass of speculators who attack as a function of the state of fundamentals.

(e) Add $a(\theta)$ to your plot. Show graphically the region where the currency is defended. As a function of x^*, compute θ^*, the critical value of fundamentals such that the government defends the currency if and only if $\theta > \theta^*$.

Parts (a) through (e) are relatively straightforward.

(f) Find θ^* and x^* in equilibrium. You will have a quadratic equation for θ^*.

Assume a fixed level of θ^* and write the expected payoff of the speculator with signal $x \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$. This speculator believes that the fundamentals are distributed uniformly on $[x - \varepsilon, x + \varepsilon]$, and if they fall in the interval $[x - \varepsilon, \theta^*]$, then the currency collapses. Plug in $x = x^*$, express x^* in terms of θ^* using (e), and equate the expected payoff of that speculator to 0. Solve for θ^* (you will have a quadratic equation).

(g) Is Theorem 2 from Morris and Shin (1998) correct?

Find if $f(\theta^*) = e^* - 2t$ when you plug in $\varepsilon=0$. If so, then Theorem 2 is correct, otherwise not.