Economics 209A
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Hints to Problem Set 3 (now due **Friday, October 14**)

Problem 1 is a relatively straightforward application of Myerson (1981). Remember that it is possible that the seller will keep the item.

Problem 2.

This is a hard problem. Consider a setting with a seller and a buyer, whose valuations v_1 and v_2 are drawn independently from the uniform distribution on [0, 1]. Given the seller's valuation, the optimal mechanism to sell the object is to post the price of $p = (1+v_1)/2$ and to sell the object only if the buyer is willing to pay that price. However, what if the seller has an opportunity to design a mechanism before he learns his valuation? Prove that the mechanism above is still optimal.

Use the methods of Myerson (1981) to derive an optimal mechanism. In a direct revelation mechanism, the buyer and the seller can announce their types simultaneously. The allocation and the transfers are functions of announcements. This type of a mechanism may potentially do better than the one proposed in the problem, because the buyer does not know the seller's type when he makes his announcement. Thus, the buyer's incentive compatibility may be easier to satisfy. However, even though potentially a general mechanism may do better, it does just as well, which is what you are required to show.

Denote by $p_1(v_1, v_2)$ and $p_2(v_1, v_2)$ the probabilities that the seller and the buyer gets the item. For this problem you can assume that the seller's can commit to tell the truth, so you can ignore the seller's truth-telling constraint (although ultimately this assumption does not alter the answer). Write the total surplus as in lecture notes. Write the buyer's expected payoff. Subtract the buyer's payoff from the total surplus to get the seller's payoff. Maximize. Show that the optimal mechanism creates the same allocation as the one suggested in the problem. Argue that the lowest type of the buyer gets the same payoff in both mechanisms. Use revenue equivalence theorem.

Problem 3.

Let c(S) be the Nash bargaining solution relative to the disagreement point (0, 0). Prove or find a counterexample to the following statement: if S is a subset of S′, then c(S′) is at least as good as c(S) to both players. If you find a counterexample, please illustrate it graphically. If you find a proof, please be concise.

There is a counterexample. Please draw a picture to show it.

Problem 4.

I changed the phrasing a little bit to avoid confusion (also see explanation).

This problem is based on Nash (1953). Consider a bargaining situation is define by a convex set B, a set of threats for each player A_1 and A_2 and a mapping $u : A_1 \times A_2 \to B$. The bargaining outcome is determined by a game, in which the players choose threats (mixed actions) $t_1 \in \Delta(A_1)$ and $t_2 \in \Delta(A_2)$, and the outcome from B is determined by the Nash bargaining solution relative to the disagreement point $u(t_1, t_2) = (u_1(t_1, t_2), u_2(t_1, t_2))$. Specifically, the payoffs from threats t_1 and t_2 are given by the point $(v_1, v_2) \in B$, which maximizes $(v_1 - u_1(t_1, t_2)) (v_2 - u_2(t_1, t_2))$. The Nash bargaining solution with threats is determined by **the equilibrium of the game, in which players get payoffs (v_1,v_2) when they choose $t_1 \in \Delta(A_1)$ and $t_2 \in \Delta(A_2)$, where (v_1, v_2) is the Nash bargaining solution relative to the disagreement point $u(t_1, t_2) = (u_1(t_1, t_2), u_2(t_1, t_2))$.**

Please note that functions u_1 and u_2 do not give payoffs! They give a disagreement point. The payoffs come from the bargaining solution relative to the disagreement point.

*If instead you look at the game with actions A_1 and A_2 and payoffs u_1, u_2 and find its mixed strategy Nash equilibrium (and perhaps treat it as a disagreement point), you are interpreting the problem **incorrectly**!*

Show that the Nash bargaining solution with threats **(v_1, v_2)** satisfies the following properties.

(a) The solution (v_1, v_2) is Pareto efficient.

The Nash bargaining solution relative to any disagreement point is Pareto efficient therefore …

(b) For each bargaining game (B, A_1, A_2, u_1, u_2) the solution $(v_1, v_2) \in B$ is unique.

A Nash equilibrium pair (t_1, t_2) may not be unique, but the bargaining (v_1, v_2) corresponding to any equilibrium is unique (which is what you need to show). The intuition behind this result is that effectively the game has a zero-sum structure (but not exactly, so you need to give a detailed argument).

(c) Consider two bargaining games (B, A_1, A_2, u_1, u_2) and (B', A_1, A_2, u_1, u_2). If (v_1, v_2), the solution of the first game, is an element of B' and $B' \subset B$, then (v_1, v_2) is also chosen from **B′**.

There was a typo (last B should be B′), which some of you discovered. For parts (c) and (d), it may be helpful to use some arguments similar to the ones you used in part (b), if you manage to figure it out.

(d) A restriction of the set of strategies available to a player cannot increase the value to him of the game.

The arguments for Problem 5 *are very similar to the arguments presented in the lecture notes. The main distinction is in the probability, with which the project is financed in the second period in case the agent reports a low cash flow in the first period.*

Problem 6.

Again, consider the setting of Bolton and Scharfstein (1990) with competitive investors, except assume this time that the agent also initially has cash $Y \in [0, 2(F - \pi_1)]$, which he can contribute to the project at time 0.

(a) Find R_1, R_2, R^1, R^2, β_1 and β_2 in the optimal contract. (Assume that the investors may be given the ability to disallow the agent to run the project even if he has enough cash to invest).

Follow the line of argument of problem 5, until you get to the break even constraint. That constraint will be different because the agent contributes cash at time 0. You will find that β_1 and R_2 will depend on Y. Check that for Y = 0 you get the formulas of problem 5.

I will not attempt to diminish your pleasure of figuring out (b) and (c) on your own.

(b) Under what conditions does a feasible contract exist?

(c) Find the manager's expected payoff as a function of Y. Compare the derivative of his payoff with respect to Y with 1. Interpret.