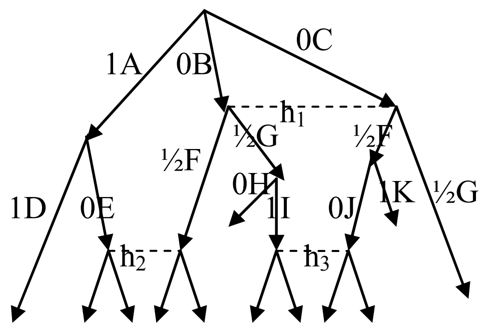
Economics 209A
Professor Yuliy Sannikov

Problem Set 2 (due September 26, 2005)

Directions: Make every attempt to do each problem on your own. If you need help, please feel free to consult papers or discuss the problems with your classmates. **On your solutions, please acknowledge any references you used or help you got, e.g. "I discussed this problem with so and so."** In your formal proofs, please be concise.

Problem 1.

Consider the following figure, which shows actions available in each information set and probabilities with which each action is taken.




(a) What is the minimal number of players in this game, for which all assumptions of Section 2 of Kreps and Wilson (1982) hold? Please justify your answer by showing how players can be assigned to nodes and information sets to ensure that all the assumptions hold, and explain why this cannot be done with fewer players. Please answer this question with a picture and a *brief* explanation.

(b) Please characterize the set of beliefs in information sets $h_1$ and $h_3$ which are consistent with the players' strategies. Please represent your answer graphically as a subset of a unit square.

(c) For each combination of beliefs on sets $h_1$ and $h_3$, what is the set of beliefs on the information set $h_2$ that is consistent with the players' strategies.

Problem 2.

Consider the following game. There is a seller and many buyers. In each period, first the seller produces an item with quality either $q = 0$ or 1. It costs 0 to produce a low-quality item and 1 to produce a high quality item. A high-quality item has value $v(1) = 3$ to the buyers and a low-quality one has value $v(0) = 1$. After the item is produced, the seller holds a first-price auction, in which buyers simultaneously bid for the item. The item goes to the highest bidder, who pays his bid. Potential buyers do not know the true quality of the item when they submit bids, but learn about the true quality at the end of the period.

(a) If the game is repeated for 1 period, what happens in a SPE?
(b) If the game is repeated for N periods, what happens in a SPE?

Now, suppose that the seller may be normal or behavioral. Normal seller can choose to produce an item of either quality, but the behavioral seller always produces an item of high quality. At the beginning the buyers hold a belief that the seller is behavioral with probability $p_1$, and the game is repeated for N periods.

(c) Characterize a sequential equilibrium of this game.

Problem 3.

Recall the setting of Problem 2 on the previous problem set. A seller is auctioning a plant, and two companies are participating in an auction. The valuations $v_1$ and $v_2$ of the two companies are drawn independently from the uniform distribution on [0, 1]. The seller owns a fraction $\lambda$ of the first company, and therefore gets $\lambda$ of its profit. If company 1 obtains the plant and pays $b_1$, then the seller's payoff is $b_1+\lambda(v_1-b_1)$. If company 2 obtains the plant and pays $b_2$, then the seller's payoff is $b_2$. The plant is worthless to the seller.

(a) Find an optimal auction. In particular, please draw a rectangular coordinate system, with valuations of bidder 1 on the horizontal axis and valuations of bidder 2 on the vertical axis, and show who obtains the item for each pair of valuations.
(b) Compute the seller's expected utility.

Problem 4.

Suppose that there is a continuum of firms in the market with types $t \in [c, d]$. A firm of type t generates cash flows x in period 1 uniformly distributed on the interval [0, t]. In period 0 the manager of the firm, who knows the firm's type t but not the future cash flows, issues debt with face value $F \in [0, \infty)$. The manager's compensation is given by

$$M = \gamma_0 V_0 - \gamma_1 \max(F - x, 0)$$

where $V_0$ is the market's perception in period 0 of the firm's expected cash flows (given publicly observable debt level F), max(F - x, 0) is the extent of bankruptcy in period 1 when cash flows x are realized, and $\gamma_0$ and $\gamma_1$ are positive constants. In period 0 the manager of each type chooses debt level F to maximize expected utility. Find the fully separating equilibrium. Specified what restrictions on parameters you imposed for this separating equilibrium.

> Note: if you are interested in the background for this problem, you may take a look at Ross, S. "The Determination of Financial Structure: The Incentive-Signalling Approach," *The Bell Journal of Economics*, Vol. 8, No. 1. (Spring, 1977), pp. 23-40.