Economics 209A
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Problem Set 1 (due September 12, 2005)

Directions: Make every attempt to do each problem on your own. If you need help, please feel free to consult papers or discuss the problems with your classmates. On your solutions, please acknowledge any references you used or help you got. For example, "I discussed this problem with so and so."

In your formal proofs, please be concise.

Problem 1.

Consider the following game related to the war of attrition discussed in class. Two players initially make demands (2,1) and (1,2) about how to split $3, and play the war of attrition. Both players discount future payoffs at the common rate r. However, unlike in the standard war of attrition, player 2 may be a behavioral type with probability $p \in (0,1)$ or a normal type with probability 1-p. The behavioral type is not able to concede. The structure of the game is common knowledge, but only player 2 knows whether he is really normal or behavioral. The purpose of this problem is to characterize all mixed strategy Bayesian Nash equilibria. Denote by F_1 the CDF of concession times of player 1, and by F_2 the CDF of concession times of the *normal* type of player 2. Denote by T the time when the play ends for sure, i.e. $T = \min_t \{F_1(t) = 1\}$..

(a) Prove that $\min_t \{F_2(t) = 1\} \geq T$
(b) Prove that F_1 and F_2 may not have atoms on (0, T].
(c) Prove that there is no subinterval in (0, T) where both F_1 and F_2 are flat.
(d) Prove that there is no subinterval in (0, T) where F_1 or F_2 alone is flat.
(e) Find all Bayesian Nash equilibria, and justify your logic.

Note: For this problem, Abreu and Gul (2000) is helpful.

Problem 2.

A seller is auctioning a small plant, and two companies are participating in an auction. The valuations v_1 and v_2 of the two companies are drawn independently from the uniform distribution on [0, 1]. The seller owns a fraction λ of the first company, and therefore gets λ of its profit. If company 1 obtains the plant and pays b_1, then the seller's payoff is $b_1 + \lambda(v_1 - b_1)$. If company 2 obtains the plant and pays b_2, then the seller's payoff is b_2. Because of this asymmetry, the seller would like to design an auction that favors company 1. Therefore, instead of comparing the bids to determine the winner, the seller chooses to compare ab_1 and b_2, where $a > 1$ is announced at the beginning of the auction. The winning company pays its bid.

(a) Find an equilibrium, in which the bidding functions are of the form

$$\sigma_1(v_1) = \begin{cases} kv_1 & v_1 \le v^* \\ kv^* & v_1 \ge v^* \end{cases} \qquad \text{for company 1 and}$$

$$\sigma_2(v_2) = lv_2 \qquad \text{for company 2,}$$

with $akv^* = l$.

(b) Compute the seller's expected payoff from this equilibrium.

(c) For what value of a is the seller's expected payoff maximized in an auction with this structure?