Costly State Verification.

Principal ← investor
Agent ← entrepreneur, who needs investment K

If investment is made, project has returns y, distributed on $[0, \bar{y}]$ with CDF F. Only the agent observes the true returns. However, the investor can verify returns at a cost c. The agent and the investor are risk-neutral. What is the optimal contract if the investors are competitive?

By revelation principle, we can restrict attention to direct revelation mechanisms:

- agent reports $y \in [0, \bar{y}]$
- investor verifies if $y \in S^V$ and does not verify if $y \in S^N = [0, \bar{y}] \backslash S^V$
- g(y) is the transfer to the investor if the agent reports y (and does not lie)

If the agent lies, without loss of generality assume that the investor extracts everything. This gives the agent the strongest incentives not to lie, but does not affect payoffs in a truth-telling contract.

Problem associated with the optimal contract: $\max \int_0^{\bar{y}} (y - g(y)) dF(y)$

s.t. $\int_{S^V} (g(y) - c) dF(y) + \int_{S^N} g(y) dF(y) \geq K$ (investor breaks even)

$g(y) \leq y$ (feasibility)

and (truth-telling)

Lemma. A feasible contract satisfies truth-telling iff for some constant D

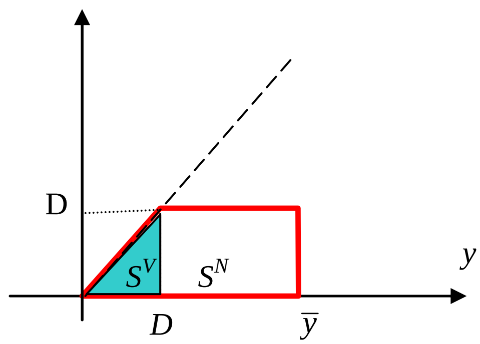
A) $g(y) = D$ for $y \in S^N$
B) $g(y) \leq D$ for $y \in S^V$

Proof. (⇒) If the agent's chooses to report in the non-verification region, he will choose a report that involves the smallest transfer. Therefore, if A) fails, it is not incentive-compatible to tell the truth in the non-verification region. Similarly, if B) fails, then there is $y \in S^V$ with $g(y) > D$. But then the agent would prefer to report something in the non-verification region than to report y. We conclude that A and B must hold in a truth-telling contract.

(⇐) If $y \in S^V$, then the agent weakly prefers to tell the truth and pay a transfer of g(y) rather than announce something in S^N and pay D or announce something else in S^V and pay y. If $y \in S^N$, then the agent is indifferent between all announcements in S^N, but weakly prefers them to any announcement in S^V.
QED

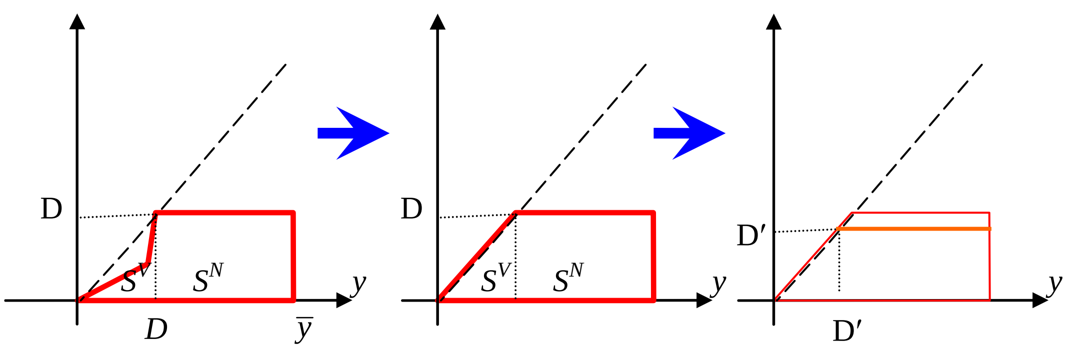
Proposition. Optimal contract is a standard debt contract, i.e. ∃ D s.t.

1) $S^V = [0, D)$ and $S^N = [D, \bar{y}]$
2) $g(y) = y$ on S^V and $g(y) = D$ on S^N



Proof. We list conditions that the optimal contract must satisfy one by one and explain why. Eventually, we arrive at the contract defined in the proposition.

(a) The investor-break-even constraint must hold with equality. If not, we can lower g(y) in all states by the same constant and improve the agent's payoff without violating incentive-compatibility.
(b) If g(y) = D, it is best not to verify y.
(c) $[D, \bar{y}] \subseteq S^N$. If not, we can find a better contract. Let us create a new contact with $S^{N\prime} = S^N \cup [D, \bar{y}]$ and $g'(y) = D$ on $S^{N\prime}$ and $g'(y) = g(y)$ on $S^{V\prime}$. Then the new contract is fully incentive-compatible and creates more total surplus. However, the agent is worse off under this contract than under the old contract. We can transfer the surplus to the agent as in (a).
(d) $g(y) = y$ on S^V. If not, we can find a better contract. Let us modify the contract by extracting y from the agent on S^V. Then the contract generates the same total surplus, but the investor gets more of the surplus. We can transfer surplus from the investor to the agent by lowering the debt level D (which saves on verification costs).



QED.