“Signaling Games and Stable Equilibria”

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Introduction

- Signaling Games often admit several Nash equilibria.

- What is a signaling game?

  One party (A) possesses private information and sends a signal to a second uninformed player.

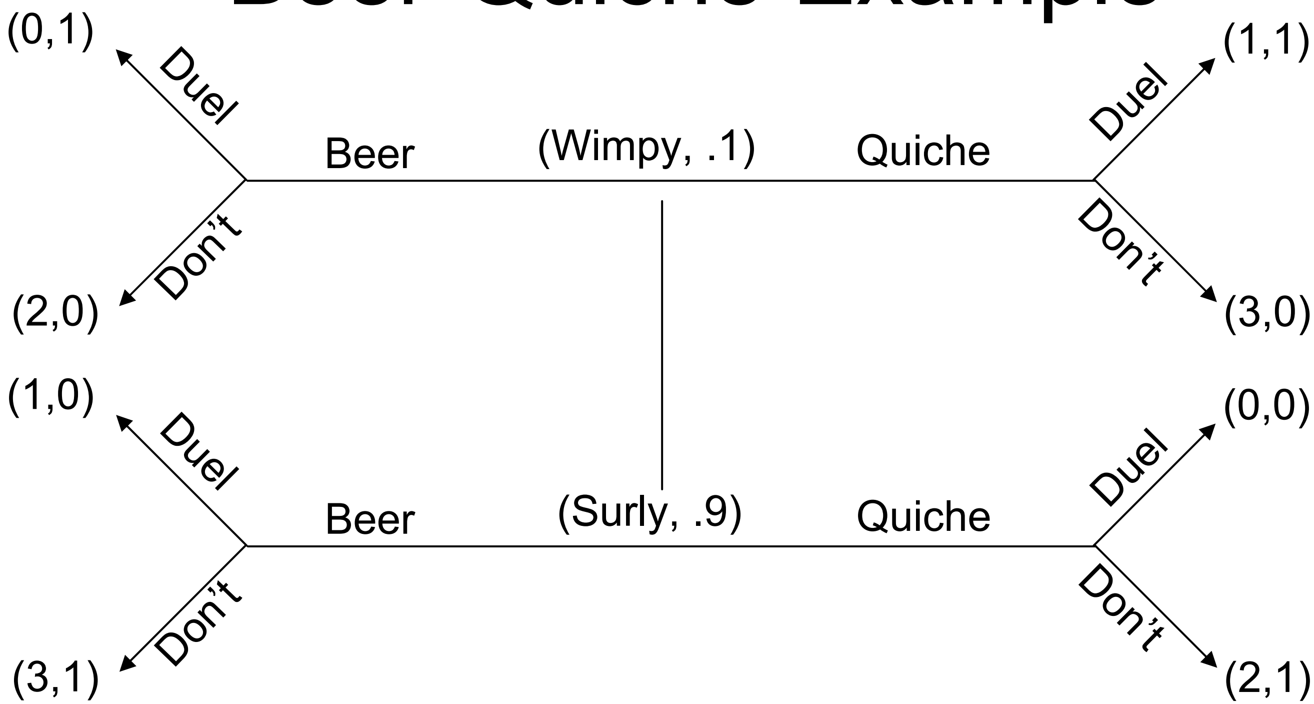
  The second player (B) receives the signal, chooses a response and payoffs of the players are realized.

- The authors show that many of the equilibria can be eliminated by restricting the out-of-equilibrium beliefs of the uninformed party in fairly intuitive ways.

Outline for Presentation

- Beer-Quiche Example
- Formulation & Preliminaries
- Equilibrium Selection Criteria
    - Intuitive Criterion
    - D1 & D2 Criteria
- Spence Job Market Signaling Example

Beer-Quiche Example

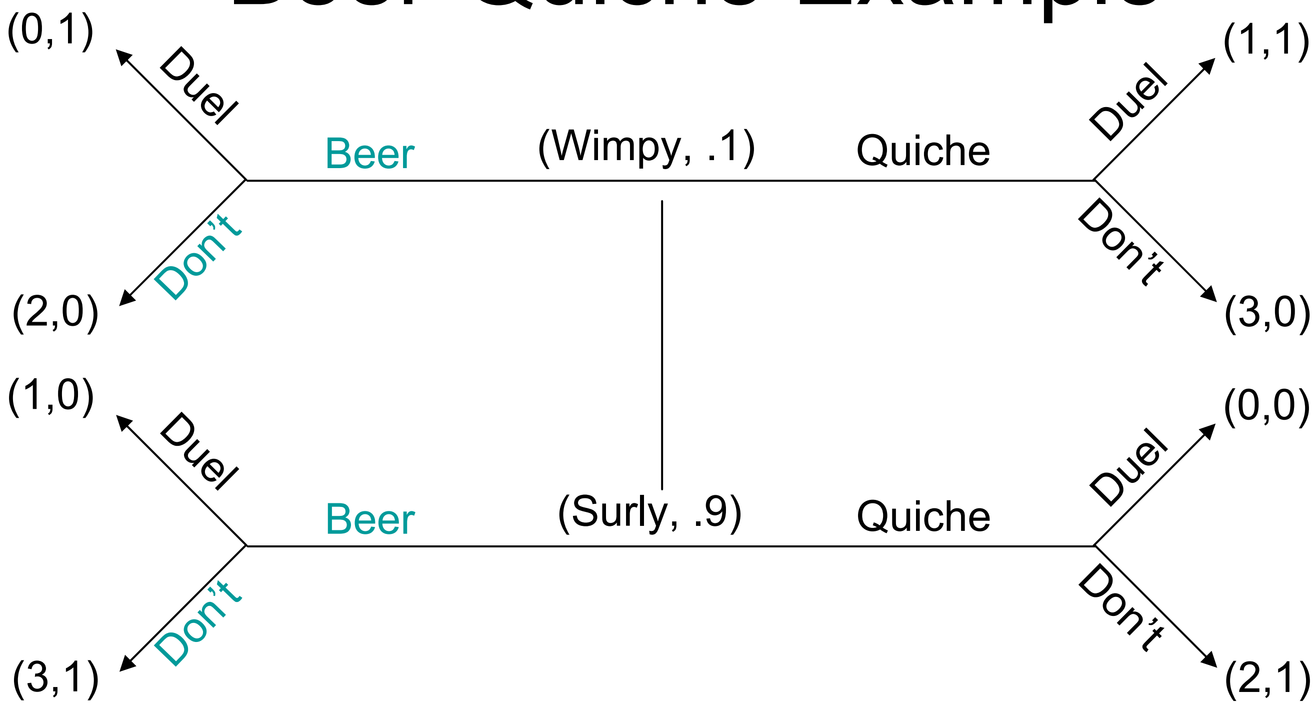


- Nash Equilibrium 1:

  A chooses beer & B chooses don't duel if he sees beer and duel with probability ≥ .5 if he sees quiche.
- Nash Equilibrium 2:

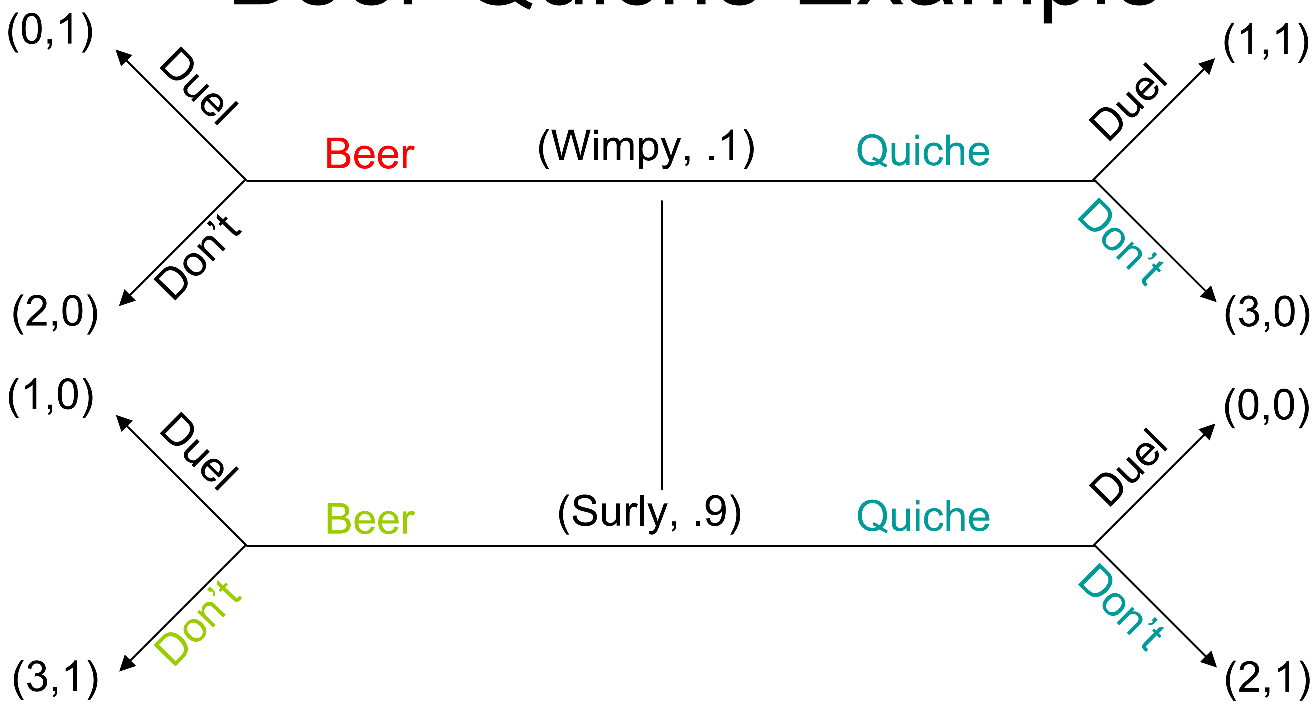
  A chooses quiche & B chooses don't duel if he sees quiche and duel with probability ≥ .5 if he sees beer.

Beer-Quiche Example



- Does equilibrium 1 make sense?
- Equilibrium 1: A chooses beer & B chooses don't duel if he sees beer and duel with probability ≥ .5 if he sees quiche.
- If B sees quiche, it is a sign that A is wimpy, so B believes A is wimpy with probability ≥ .5.

Beer-Quiche Example



- Does equilibrium 2 make sense?
- Equilibrium 2: A chooses quiche & B chooses don't duel if he sees quiche and duel with probability ≥ .5 if he sees beer.
- A wimpy A would not choose beer, while a surly A might.
- We can restrict B's beliefs so that a wimpy A will not choose beer. If he sees beer, B believes that A is surly and will choose not to duel.
- If A knows that B knows this, A would choose beer when surly.
- Equilibrium 2 breaks down!

Formulation & Preliminaries

- Player A: Informed Player

  types $t \in T$ drawn according to π

  learns type, sends message $m \in M(t)$

  $T(m) =$ set of types with m available

  behavioral strategy $\rho(m;t)$

- Player B: Uninformed Player

  receives message m & chooses response $r \in R(m)$

  $$BR(\mu, m) = \arg\max_{r \in R(m)} \sum_{t \in T(m)} v(t,m,r)\mu(t)$$

  $\mu(t) =$ posterior probability assessment

  behavioral strategy $\phi(r,m)$

- Payoffs

  player A : $u(t,m,r)$

  player B : $v(t,m,r)$

Formulation & Preliminaries

- Stability

  $\Gamma =$ class of games, $\Sigma =$ space of strategy profiles, Nash correspondence, $N : \Gamma \to \Sigma$

  For $\gamma \in \Gamma$, $M \subset N(\gamma)$ is stable if

  $\forall \varepsilon > 0, \exists \delta > 0$ s.t. every γ' within δ of γ has some Nash equilibrium less than ε from M

- Weak Best Response:

  A strategy for a player is never a weak best response relative to the set of equilibria if in equilibrium, the strategy in question is worse for the player.

- Fact 5:

  Pruning a given player's pure strategy that is never a weak best response does not affect the stability of the set of equilibria.

Equilibrium Selection Criteria

- General Idea

  Step 1: Pose some criterion for saying that a particular out-of-equilibrium message cannot “reasonably” be expected to be sent by a particular type.

  Step 2: For each out-of-equilibrium message, are B’s responses in the original game sequentially rational given that B’s beliefs are restricted based on Step 1?

1. Elimination of Type-Message Pairs by Dominance:

   For out-of-equilibrium message m, type t can be eliminated if there is some other message n such that

$$\min_{r \in R(n)} u(t,n,r) > \max_{r \in R(m)} u(t,m,r)$$

Equilibrium Selection Criteria

2. The Intuitive Criterion

Let $u^*(t)$ denote A's equilibrium payoff when his type is t.

For each out-of-equilibrium message m, form the set *S(m)*

consisting of all types such that $u^*(t) > \max_{r \in BR(T(m),m)} u(t,m,r)$.

If for any one message m $\exists\, t' \notin S(m)$ such that

$$u^*(t') < \min_{r \in BR(T(m) \setminus S(m),m)} u(t',m,r),$$

then the equilibrium fails the Intuitive Criterion.

Equilibrium Selection Criteria

3. D1 & D2 Criteria

For a given out-of-equilibrium message m, form

$$D_t = \{\phi \in MBR(T(m), m) : u^*(t) < \sum_r u(t, m, r)\phi(r)\}$$

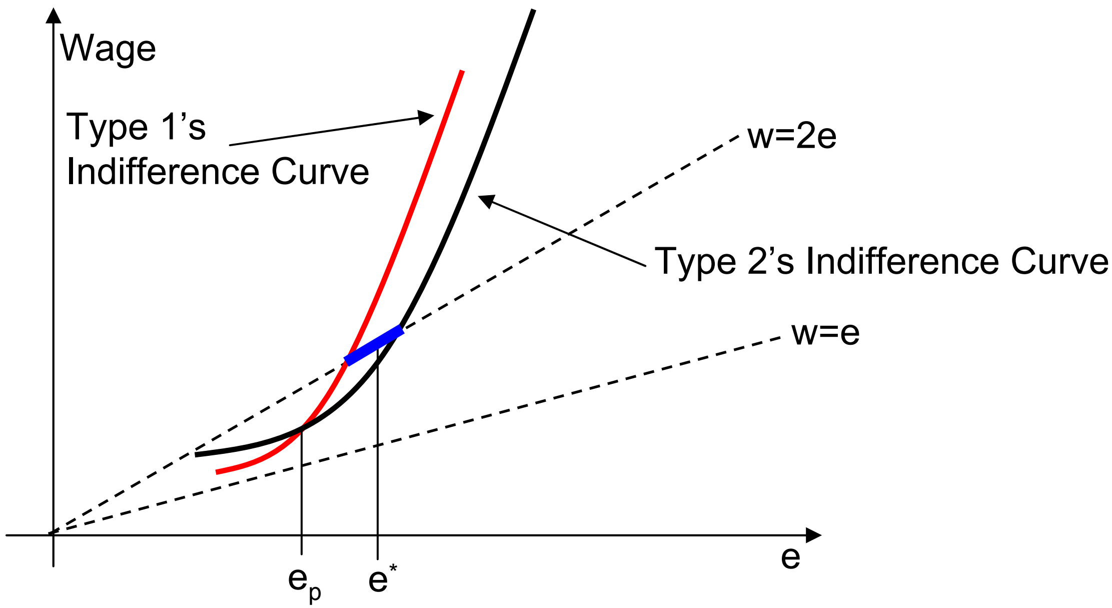
$$D_t^0 = \{\phi \in MBR(T(m), m) : u^*(t) = \sum_r u(t, m, r)\phi(r)\}.$$

Criterion D1: If for some type t there exists a second type t' with $D_t \bigcup D_t^0 \subseteq D_{t'}$, then *(t,m)* can be pruned from the game.

Criterion D2: If for some type t $D_t \bigcup D_t^0 \subseteq \bigcup_{t' \neq t} D_{t'}$, then *(t,m)* can be pruned from the game.

Spence Signaling Example

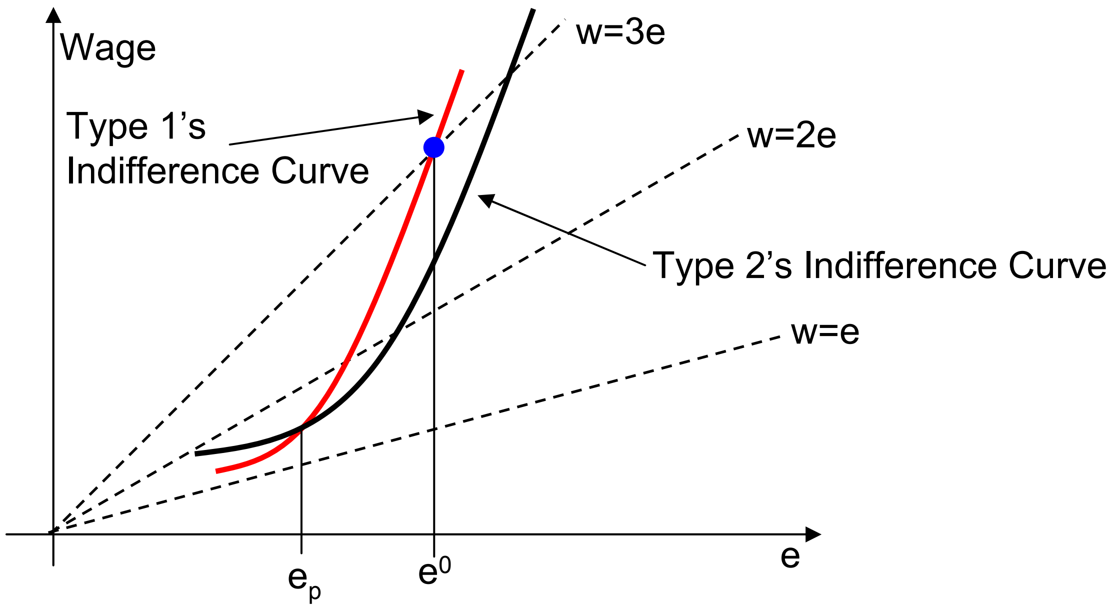
Case A: Two Types



- Does the pooling equilibrium at e_p make sense?
- For the out-of-equilibrium message e*, the set of types that is better off at e_p consists of Type 1 only. Type 2 is strictly worse off sending e_p compared to e*.
- The pooling equilibrium fails the Intuitive Criterion!
- The only equilibrium that will pass the Intuitive Criterion is the least-cost separating equilibrium.

Spence Signaling Example

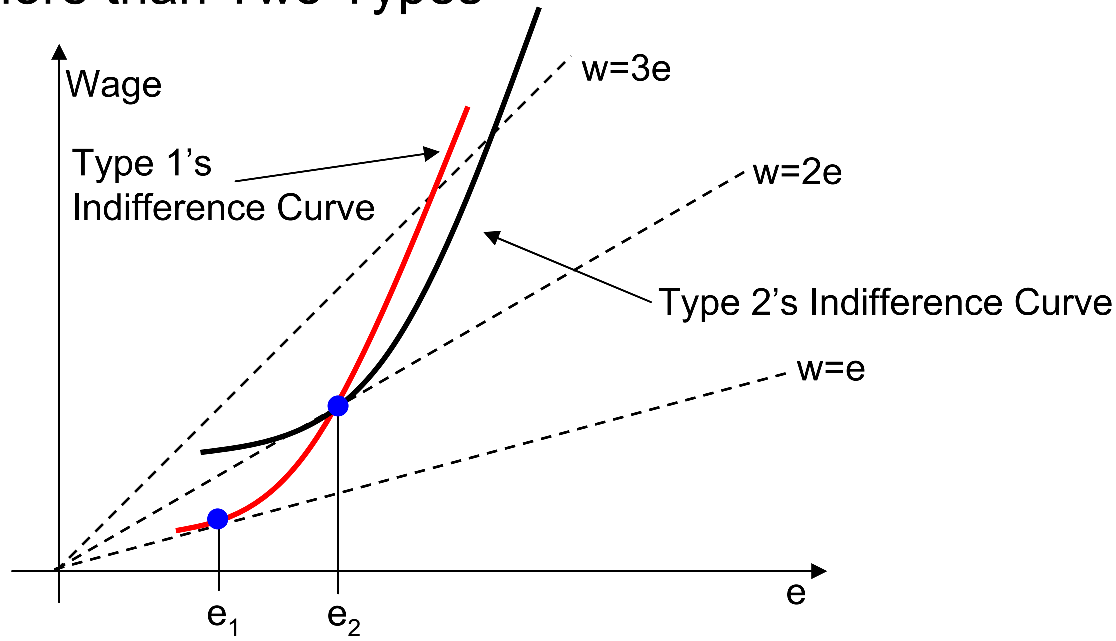
Case B: More than Two Types



- Does the pooling equilibrium at e_p make sense?
- If firms believe the out-of-equilibrium message is coming from Type 3, then they will require education e^0 to get a wage higher of 2e or higher, but this does not guarantee that Type 2 will be better off than the pooling equilibrium.
- The pooling equilibrium can pass the Intuitive Criterion!

Spence Signaling Example

Case B: More than Two Types



- Pooling equilibria can be ruled out using the D1 Criterion!
- According to D1, we can prune all type-message pairs in which each type gets less than the wage appropriate for his type.
- Therefore, each type gets a wage appropriate for at least his type. No pooling equilibrium can possess this characteristic.
- The only equilibrium that passes the D1 Criterion is the least-cost separating equilibrium!