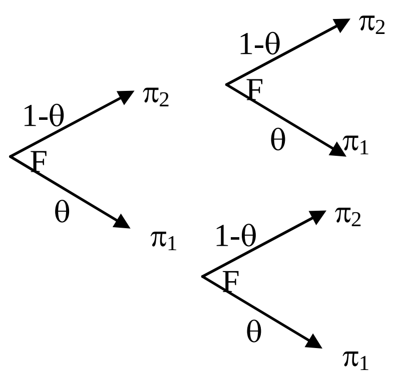
Bolton and Scharfstein (1990)

A manager has an opportunity to operate the firm for 2 periods, but needs financing from outside investors. In each period, if the firm gets F of outside funding, it gets cash flows π_1 with probability θ and π_2 with probability $1 - \theta$. Cash flows are iid over time. Thus, the potential structure of investment and cash flows looks like this:



Only the firm manager and not the outsiders see the true cash flows. Assume that $\pi_1 < F < \pi_2$ and π_1 is a verifiable component of cash flows (that is, investors can force the firm to pay π_1). The residual $\pi_2 - \pi_1$ is the non-verifiable component, which the manager can divert for personal consumption. There is no discounting between periods. Assume that

$$\bar{\pi} = \theta\pi_1 + (1-\theta)\pi_2 > F \,, \qquad (*)$$

so the project would be profitable in expectation if cash flows were verifiable. With unobservable cash flows financing would be infeasible if there was only one period, because at the end of the period investors can extract at most $\pi_1 < F$ from the manager. With two periods some financing may be feasible if the second-period financing is contingent upon the outcome at the end of the first period.

The investor's problem.

By the *revelation principle* we can restrict attention to direct-revelation mechanisms, in which the transfers from the manager to the investors and the probability of continued financing depends on the manager's truthful report. Such a mechanism is defined by

- R_i, the payment the manager makes at the end of period 1 if he reports π_i
- β_i, the probability of continued financing in the second period if the report is π_i
- R^i, the payment at the end of period 2 if the manager reports π_i in **period 1**

Note: truth-telling implies that the second-period payment at the end of period 2 cannot depend on the manager's report in period 2.

Assume that all the bargaining power belongs to the investors, so their problem is to maximize profit subject to relevant constraints (including the individual rationality constraint for the manager). The problem of finding an optimal contract can be written as follows:

$$\max_{R_i,\beta_i,R^i} -F+\theta[R_1+\beta_1(R^1-F)]+(1-\theta)[R_2+\beta_2(R^2-F)]$$

s.t. $\pi_2 - R_2 + \beta_2(\overline{\pi}-R^2) \geq \pi_2 - R_1 + \beta_1(\overline{\pi}-R^1)$, (IC2)

(IC1), $\pi_i \geq R_i$, $\pi_1 \geq R^i$.

The last four constraints (for i = 1,2) say that the investor cannot extract more cash from the agent than he reports. Because the investors cannot extract more cash than the agent has, the individual rationality constraint (that the manager's utility is at least 0) does not bind. A contract that satisfies all of the above constraints is called a *feasible*. We derive the optimal contract assuming that (IC1) does not bind, and then verify that it holds.

Derivation.

Step 1. (IC2) is binding in the optimal contract. If not, we would be able to increase R_2 or R^2, and improve the investor's profit without violating any of the constraints.

Q. If $R_2 = \pi_2$ and $R^2=\pi_1$, then we cannot increase neither R_2 nor R^2. Does the above argument fail in this case?

Step 2. There exists an optimal contract with $R^1 = R^2 = \pi_1$. If we had an optimal contract with $R^i < \pi_1$, then we could increase R^i to π_1 and decrease R_i by β_i (π_1-R^i) to get a payoff-equivalent contract, in which all the constraints are still satisfied.

Q. Verify that all the constraints are still satisfied.

Step 3. $\beta_1 = 0$ in the optimal contract. If we had a contract with $\beta_1 > 0$, then by reducing β_1 to 0, we get an improved feasible contract.

Step 4. $R_1 = \pi_1$ in the optimal contract. If not, we can increase both R_1 and R_2 by the same amount until $R_1 = \pi_1$, and improve the contract.

Q. How do we know that R_1 reaches π_1 before R_2 reaches π_2?

Last step. $\beta_2 = 1$. If we had $\beta_2 < 1$, let us increase β_2 to 1 and decrease R_2 appropriately to maintain the left hand side of (IC2) unchanged. Assumption (*) implies that more total surplus is generated when β_2 is increased to 1. The increase in the surplus should go entirely to the investors, because the manager's expected payoff remains unchanged.

Conclusion. In the optimal contract, $R_1 = R^1 = R^2 = \pi_1$, $\beta_1 = 0$ and $\beta_2 = 1$. R_2 is determined by (IC2) to be $\overline{\pi}$.

Note that (IC1) holds, because an agent with low cash flows cannot pretend to have high cash flows.

Q. Under what conditions is there a contract which gives the investor nonnegative profit?
Q. Does this contract require commitment on the part of the investor?

Discussion.

- There is *ex-post inefficiency*.
- The contract is renegotiation-proof.
- The firm's performance affects its financing costs and future access to capital.

Two reasons why staged financing is natural:

- reduces adverse selection problems: entrepreneurs who have confidence in the venture accept contracts more willingly
- staged financing limits the extent to which management can pursue its own self-interest (e.g. steal cash, as in this model)