# Abreu, Pearce and Stacchetti, "Optimal Cartel Equilibria with Imperfect Monitoring," *JET* 1986.

Asaf Plan*

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*asaf@econ.berkeley.edu

The Question

Consider collusion among identical quantity-setting oligopolists, who cannot directly observe each others' past quantity choices, but only past market prices, which are partly random.
(The Green and Porter [1984] model.)

- What are the optimal cartel equilibria?

The Difficulty

- The game is infinitely repeated, and

- Each firm's strategy at each point in time potentially depends on the entire past price history and it's entire past own quantity history.

That's hard — or so it seems...

## The Previous Approach

Porter (1983) considers the problem of maximizing equilibrium cartel member profits under a limited set of strategies based on trigger prices and Cournot-Nash reversion. Specifically,

- Each firm produces the cartel quantity until the market price drops below a certain trigger price,

- Then each firm would "punish" by increasing its quantity to the Cournot-Nash level, for $T$ periods.

However this turns out not to be optimal over a broader set of strategies.

The Authors' New Approach

The authors are able to find optimal pure strategy symmetric sequential equilibria (SSE), without restrictions on strategies, by

- Assuming firms' possible quantity choices are discrete, and
- Reducing the repeated game to an equivalent static game ( "strong flavor of dynamic programming" ).

The Authors' New Approach, continued

"One can imagine constructing a new game by truncating the discounted supergame as follows: after each first-period history, replace the SSE successor by the payoffs associate with that successor.
The first-period equilibrium quantities will still constitute an equilibrium of the new game, and the resulting total payoffs will also be the same." (p. 253)

The Model

The single-period game:

- $N$ identical firms simultaneously choose quantities of output to produce, $q_i \in \mathbb{N}$ (discrete!), $i = 1, ..., N$,

- Firms incur total production cost $c(q_i) \geq 0$, and

- Market price $p$ is a random variable whose distribution depends on aggregate production $q = \sum q_j$, with density $g(p; q)$.

The Model, continued

Thus the expected payoff is

$$\bar{\pi}_i(q_1, ..., q_N) = \mathsf{E}[p|q]q_i - c(q_i)$$

And the one-period game is given by

$$G = (S_1, ..., S_N; \bar{\pi}_1, ..., \bar{\pi}_N),\ S_i \in \mathbb{N},\ i = 1, ..., N$$

The Model, continued

The repeated game:

$G^{\infty}(\delta)$ is the infinitely repeated game defined by the stage game $G$ and the discount factor $\delta \in (0, 1)$.

A firm's strategy specifies an output in each period as function of past prices and the firm's own past quantities.

Sketch of Reduction to a Static Game

Consider the truncated game in which players first play the stage game and then additionally receive some symmetric payment depending on the price that arose in the stage game.

Consider a bounded set of possible additional payments, $W \subset \mathbb{R}$. Let $B(W) \subset \mathbb{R}$ represent the total payoffs that players could receive in pure strategy equilibria of the truncated game.

$W$ is said to be *self-generating* if $W \subset B(W)$.

Sketch of Reduction to a Static Game, continued

Let $V$ be the set of payoffs resulting from SSE's in the repeated game.

- Every self generating set is contained in $V$ (Proposition 1).

- $V$ is itself self-generating (Proposition 2).

- $B(W) = B(\{\min W, \max W\})$ (Proposition 3). (Analogous to bang-bang theorems?)

- $V$ is compact (Proposition 4), so $\bar{v} = \max V$ and $\underline{v} = \min V$ exist.

Equilibria

Consequently every payoff associated with an SSE can be supported by an SSE which in each contingency apart from the first period looks like the first period of either the best or the worst SSE.