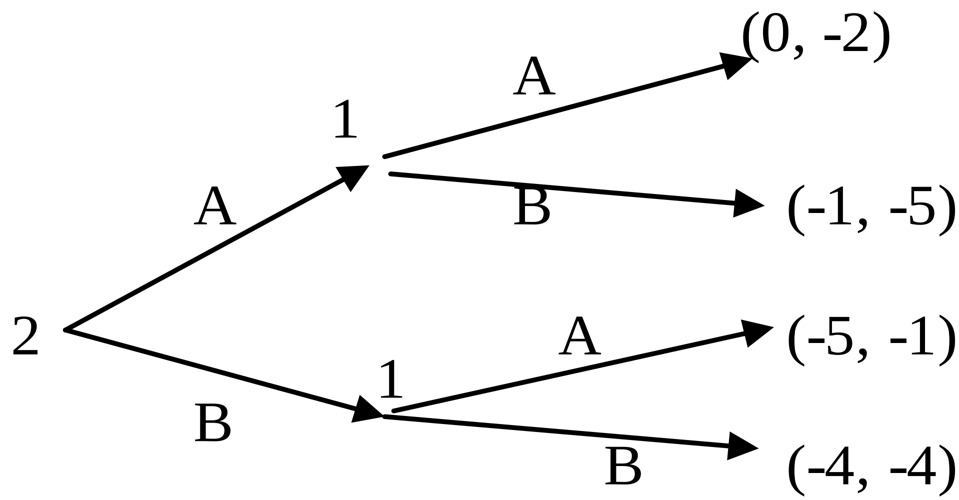
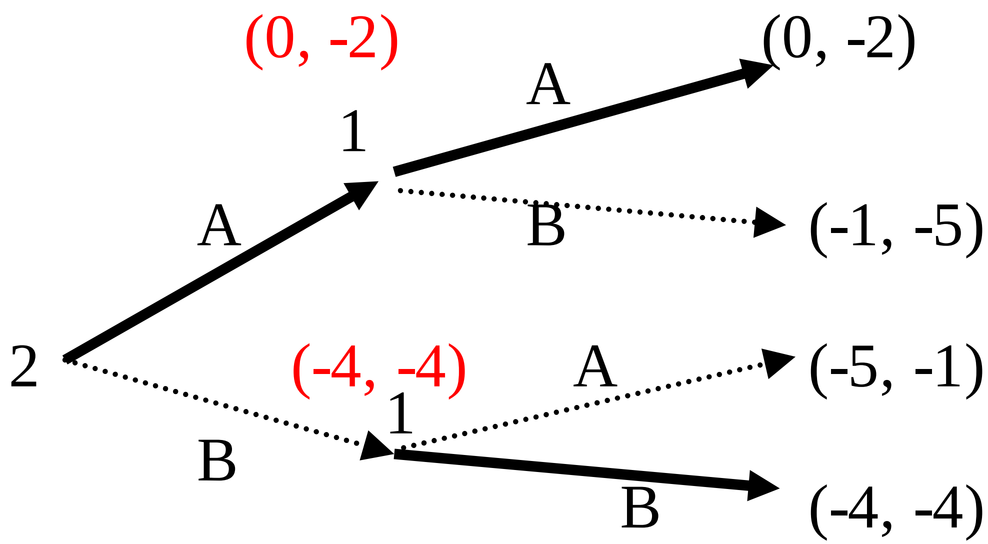
Extensive Form Games: Perfect Information

MWG 7A, B, C; 9A, B

	A	B
A	0, -2	-5, -1
B	-1, -5	-4, -4



Definition. A *strategy* is a complete plan of actions of a player. In an extensive form game with perfect information, a *strategy* specifies an action of a player in each node.

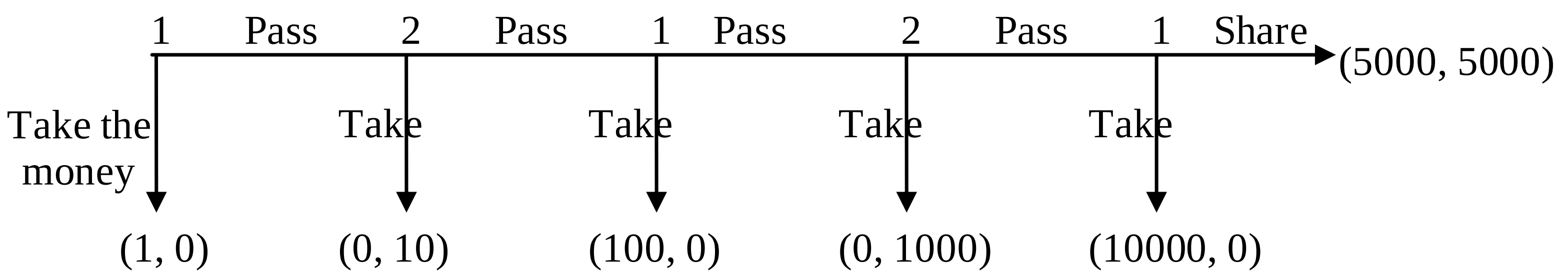


Backward induction $\rightarrow$ subgame perfect equilibrium

Informal Definition: A *subgame perfect equilibrium* (SPE) is a combination of strategies, one for each player, such that each player chooses his action optimally in each node anticipating what other players will do in all subsequent nodes of the game tree. A *subgame* is a game that follows after a given node has been reached.

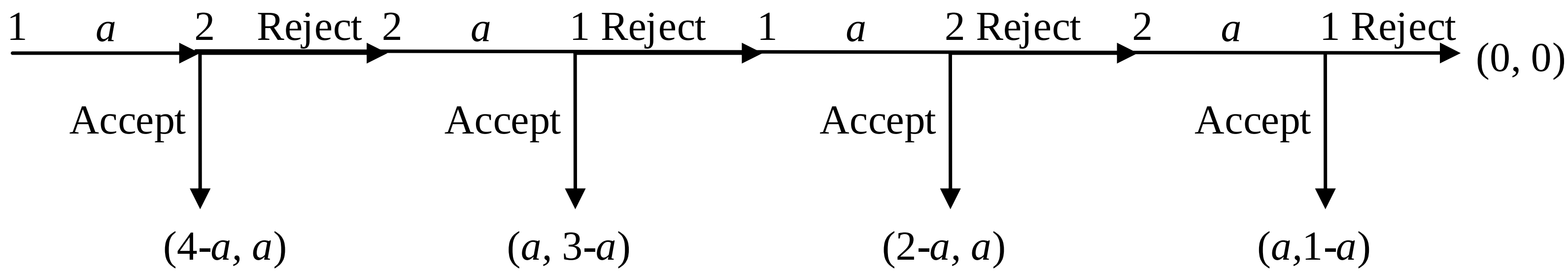
Remark: If a pair of strategies is a SPE, they form an SPE in each subgame also.

Example: Centipede game

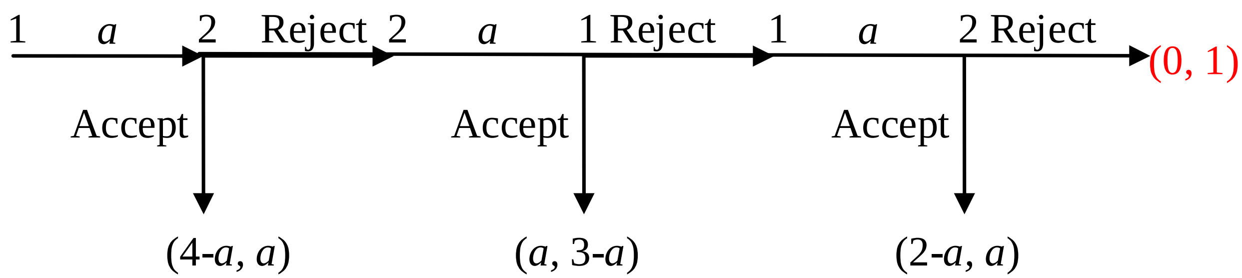


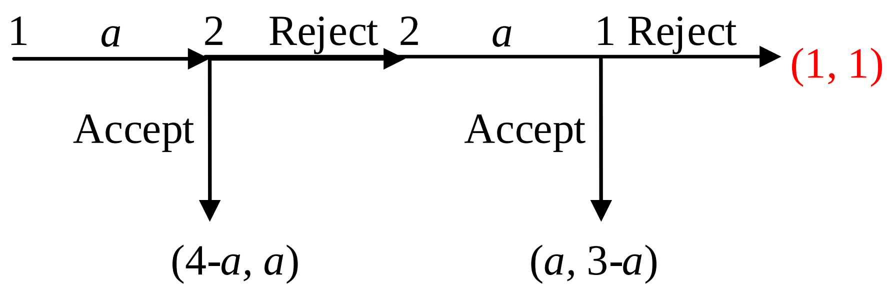
Exercise: solve this game by backward induction.

Rubinstein's Bargaining Game:



Rubinstein's Bargaining Game:

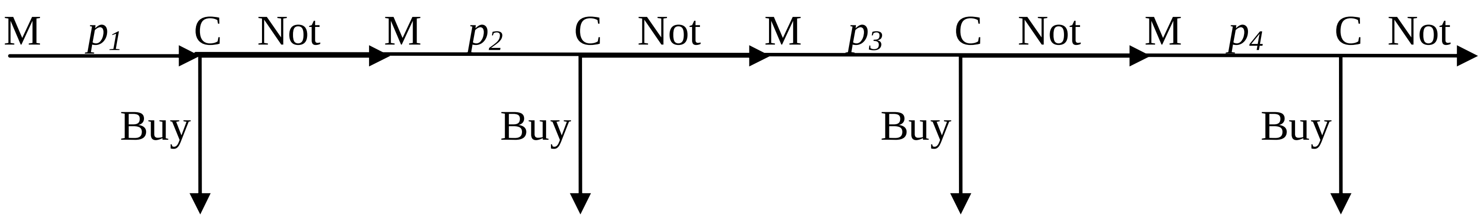




Exercise: Finish solving the game.

Durable Goods Monopoly:

- Monopolist with zero marginal cost
- Customer: valuations uniformly distributed on [0,1]
- 4 periods, timing is like this:



- if a customer with valuation $v \in [0,1]$ buys in period t, he gets $(5-t)v - p_t$
- Find SPE

- Given any sequence of prices, if a customer with valuation v buys at time t, then any customer with valuation $v' > v$ will buy at time t or earlier

- In equilibrium in any period t, customers with valuations $[0, v_t]$ are left, with $v_1 = 1$

- In the last period the monopolist will choose p_4 to maximize $p_4(v_4 - p_4)$, so $p_4 = v_4/2$ and last period profit is $v_4^2/4$.

- In period 3, price is p_3 and customers $[v_4, v_3]$ buy. Customer v_4 can be found as follows: he is indifferent between buying in periods 3 and 4, i.e.

$$2v_4 - p_3 = v_4 - p_4 \Leftrightarrow 2v_4 - p_3 = v_4 / 2 \Rightarrow v_4 = 2/3 p_3$$

- profit in period 3 is $p_3 (v_3 - 2/3p_3)$
- M chooses p_3 to maximize $p_3(v_3 - 2/3p_3) + v_4^2/4 \Rightarrow p_3 = 0.9v_3$
- profit in periods 3 and 4: $p_3 v_3 - 5/9 p_3^2 = 9/20 v_3^2$

- Moving on: customer v_3 must be indifferent between buying in periods 2 and 3, i.e.

$$3v_3 - p_2 = 2v_3 - p_3 \Leftrightarrow 1.9v_3 = p_2 \Rightarrow v_3 = 1/1.9 p_2$$, etc.

Simultaneous Moves, Imperfect Information and Information Sets

Example: finitely repeated Prisoners' dilemma

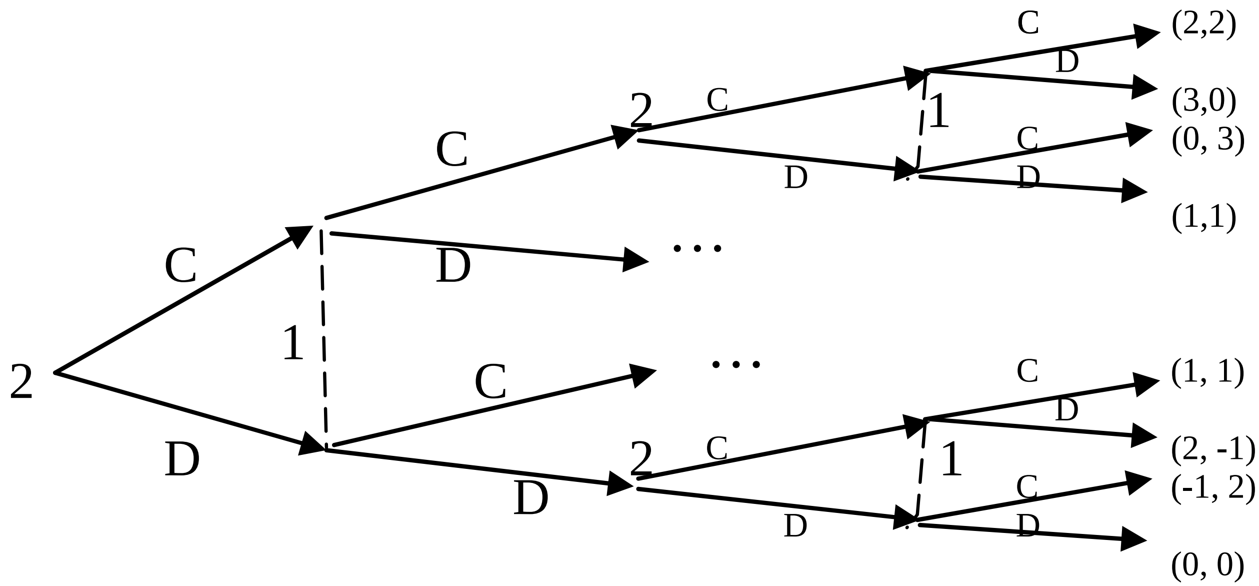
	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

Exercise: try to solve by backward induction

Tree representation of twice repeated Prisoners' dilemma



If several nodes are connected with a dashed line, those nodes are in the same *information set*. When a player arrives in an information set, he does not know which node he is in.

Formal definitions

A *subgame* is a subset of the game such that

- it begins with an information set that has one node
- it contains all successor nodes and no other nodes
- it contains only whole information sets

A *subgame perfect equilibrium* is a combination of strategies, one for each player such that in every *subgame* the strategy of each player is a best response to the combination of his opponent's strategies.

Prisoners' dilemma finitely repeated *n* times:

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→ . . .

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

Exercise: what is the subgame perfect equilibrium?

Prisoners' dilemma repeated twice and coordination:

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	A	B
A	1, 1	0, 0
B	0, 0	10, 10

Exercise: is there a subgame perfect equilibrium with cooperation in the Prisoners' dilemma?

Example: Prisoner's dilemma and Battle of the Sexes

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

Exercise: Is it possible to have cooperation in the first period?

<u>Homework Problem 2</u>: Prisoner's dilemma repeated *n* times and the Battle of the Sexes

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→ • • •

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

→

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

Consider pure strategy SPE of the game above. Is it possible to have cooperation in the first period? What is the maximal number of periods that the players can cooperate? Specify a SPE in which the players cooperate for the maximal number of periods, that is, specify their actions in every single subgame.