

Final Exam

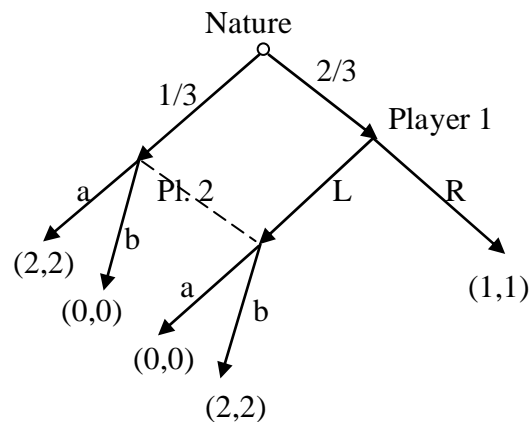
Instructions: The exam is open book and open notes. Please answer all four questions. You have three hours to complete the exam. Please be clear but concise.

PLEASE BOX YOUR ANSWERS.

Good luck!!!

Problem 1. (20 points) In the game below, please find all Perfect Bayesian Equilibria. Please specify all strategies and beliefs clearly in the following format:

	Player 1's strategy	Player 2's strategy	Player 2's belief
Equilibrium 1			
Equilibrium 2			
Equilibrium 3			



Problem 2. (25 points) Consider the following education signaling environment. There are 2 types that have cost of education $c_L(e) = e^2$ and $c_H(e) = ae^2$, where $a \in (0, 1)$. The productivities of the two types are $1 + e$ and $2 + e$ respectively.

- (a) What education level will type L choose in any separating equilibrium?
- (b) What is the smallest level of education that the high type must take in any separating equilibrium?
- (c) What is the efficient education level for type H, as a function of a ? The efficient education level maximizes productivity minus the cost of education.
- (d) Consider the unique separating equilibrium that satisfies the intuitive criterion. Please find the interval of values of a , for which type H chooses the efficient level of education in this equilibrium? Hint: compare your answers for parts (b) and (c).

Problem 3. (25 points) There is a seller with one item and two potential buyers whose valuations are independently distributed according to a uniform distribution on $[0, 1]$. An auction proceeds by the following rules. First, buyer 1 secretly submits a bid $b \geq 0$ to the seller. Upon seeing the bid of buyer 1, the seller can make any take it or leave it offer p to buyer 2. If buyer 2 accepts, the item goes to buyer 2 for the price of p . If buyer 2 rejects, the item goes to buyer 1 for the price of b (the amount of 1's bid).

(a) If the seller offers price p to buyer 2, with what probability will buyer 2 accept the offer? Please write the seller's expected payoff as a function of b and p . What offer will the seller make to buyer 2 in equilibrium, as a function of buyer 1's bid?

(b) If buyer 1 bids b , with what probability will the seller come back to him in equilibrium? What is buyer 1's expected payoff, as a function of his valuation v_1 and his bid b ? What is buyer 1's equilibrium bidding function?

(c) Compute the seller's expected revenue.

Problem 4. (30 points) Consider the following game. There is a seller and a buyer. First, the seller produces an item with quality either $q = 0$ or 1 . It costs 0 to produce a low-quality item and 1 to produce a high quality item. A high-quality item has value $v(1) = 3$ to the buyer and a low-quality one has value $v(0) = 1$. The buyer, who cannot observe the quality of the product, can offer a price p from among the following three: 0.5 , 1.5 or 2.5 . The seller can accept or reject. If the seller accepts, his payoff is $p - q$ and the buyer's payoff is $v(q) - p$. If the seller rejects, his payoff is $-q$ and the buyer's is 0 .

(a) Draw the game tree.

(b) Find a subgame perfect equilibrium. Please mark the seller and the buyer's equilibrium actions in all nodes of the tree and specify what payoff each player gets in equilibrium.

Now suppose that this game is repeated infinitely many times with the same seller and a different potential buyer in each period. The new buyer in each period knows the quality of all previously sold items. If the item is not sold in a given period, it goes bad and the seller needs to produce a new item in the following period. The seller's discount factor is δ . Note that the repetition of a SPE from part (b) constitutes a SPE of the repeated game. This SPE is not ideal for the seller. Ideally, the seller would like to produce a product of high quality in every period and be offered a price of 2.5 .

(c) Suppose that for some reason the buyers always offered price 2.5 if the seller never sold a low-quality good previously, and price 0.5 otherwise. Under what conditions on δ will the seller produce a high-quality good in every period? Hint: Under these conditions if the seller always produces a high-quality good, he gets a price of 2.5 in every period. If he deviates once, he gets a price of 2.5 for a low-quality good in one period, and price 0.5 thereafter.

(d) In reality, the buyers can also offer a lower price and the seller must commit not to accept any price lower than 2.5 . For what values of δ is there a SPE in which the seller produces a product of high quality in every period and gets the price of 2.5 , but if a buyer deviated and offered any lower price, the seller would reject it? Please find this range of δ and box it.