Econ 201A. Part II.
Due: Tuesday, December 13

<u>Problem Set 3</u>

Homework policy: Please try to do the following problems on your own first. If you get really stuck, feel free to discuss them with other people in the class, but acknowledge any discussion or ideas that you get on your homework, e.g. "I benefited from discussion with so-and-so on problem x." Please write your solutions clearly and concisely.

<u>Problem 1</u>. Consider the symmetric auction environment discussed in class. There are N bidders with valuations uniformly distributed on the interval $[0, \alpha]$. Consider an all-pay auction: the highest bidder wins, and all bidders pay their bid. Please find the equilibrium bidding function. Please compute the seller's expected revenue and compare it with that from the first price auction and a second price auction.

<u>Problem 2:</u> (Similar to Kreps, Problem 2 in Chapter 17). Consider the following signaling environment from class: 2 types that have cost of education $c_L(e)$ and $c_H(e)$ respectively with $c_L' > c_H' > 0$, $c_L'' > 0$ and $c_H'' > 0$. The productivities of the two types are $x_L + e$ and $x_H + e$. Assume that $c_L'(0) < 1$ and $c_H'(0) < 1$, but $c_L'(e) \to \infty$ and $c_H'(e) \to \infty$ as $e \to \infty$.

(a) Are there separating equilibria where one type (or both) chooses more than one level of education? If they exist, do they satisfy the intuitive criterion? By a separating equilibrium we mean that if one type chooses a given education level with positive probability, then the other type does not.
(b) Is there any pooling equilibrium where both types choose more than one level of education? By a pooling equilibrium we mean that every education level chosen by one type with positive probability is chosen by the other type with positive probability.
(c) A hybrid equilibrium is one in which some education levels are chosen by one type only and others are chosen by both types. Are there any hybrid equilibria?

<u>Problem 3.</u>

Consider the following model of initial public offerings. There is an entrepreneur who would like to take his company public. He has private information about the future profits of the company, which are either high (equal to 2) or low (equal to 1). The market's prior belief is that $\theta \in \{1, 2\}$ is equally likely to be high or low.

The entrepreneur chooses the fraction $q \in [0, 1]$ of the company to sell to the market. The market observes q and forms a belief about the firm's future profits. If the market assigns belief $\mu(q)$ that the company will have high profits, then the offering price per share will be:

$$p(q) = \mu(q) \cdot 2 + (1 - \mu(q)) \cdot 1.$$

If the true profitability is θ, the entrepreneur offer q for sale, and the market pays a per-unit price p, the market's gain is $\theta \cdot q - p \cdot q$, while the entrepreneur's utility is $p \cdot q + \frac{\theta}{2} \cdot (1 - q)$.

We start by looking for separating equilibria (q_L, p_L), (q_H, p_H) where e.g. when $\theta = 2$, the entrepreneur offers a fraction q_H to the public and the price is p_H.

(a) Show that in any separating equilibrium, $q_L = 1$ and $p_L = 1$.

(b) Derive conditions on q_H, p_H such that (q_H, p_H) could be part of a separating equilibrium where the entrepreneur choose q_H when $\theta = 2$.

(c) If $(q_L, p_L), (q_H, p_H)$ is a separating equilibrium, what must be true of $p(q)$ for all $q \notin \{q_L, q_H\}$ in equilibrium.

(d) What is the most efficient separating equilibrium? Show that it is the only separating equilibrium that survives the Intuitive Criterion.

Problem 4.

Consider the same IPO model as above, but now consider pooling equilibria.

(a) Show there is a pooling equilibrium in which $q = 1$ and $p = 3/2$.

(b) For what other values of q is there a pooling equilibrium? Characterize these equilibria.

(c) Show that any pooling equilibrium fails the Intuitive Criterion.

Problem 5.

Consider the following sequential auction model. At date 0, nature chooses values $v_1, v_2 \in [\underline{v}, \overline{v}]$ independently from a distribution F and reveals value v_i to bidder i. At date 1, bidder 1 chooses a bid $b_1 \in [0, \infty)$. This bid is observed by bidder 2, who responds with a bid $b_2 \in [0, \infty)$. The player with the highest bid wins the auction (with player 2 winning ties) and pays her bid. So $u_i = v_i - b_i$ if i wins and $u_i = 0$ otherwise.

(A) Show that in any perfect Bayesian equilibrium, bidder two must use the strategy.

$$b_2(v_2) = \begin{cases} b_1 & \text{if } b_1 \leq v_2 \\ < b_1 & \text{if } b_1 > v_2 \end{cases}$$

(B) Use your answer to (A) to solve for the perfect Bayesian equilibrium of the auction.

(C) Compute the seller's expected revenue and compare it to that in the standard first price and second price auctions.

Problem 6. **(OPTIONAL)**

Consider the following version of the Prisoners' Dilemma with asymmetric payoffs:

	C	D
C	3, 2	-1, 3
D	5, -1	0, 0

This game is repeated infinitely with discount factor δ.

A. What are the payoffs in the worst possible subgame perfect equilibrium? Prove that it is impossible to achieve payoffs worse than those.

B. What is the lowest discount factor δ^* for which the players achieve cooperation in a SPE?

C. Is it possible to achieve payoffs better than (0, 0) in a SPE for discount factors lower than δ^*? Hint: Think about a SPE in which players alternate between (C, D) and (D, C) on the equilibrium path.

Problem 7. **(OPTIONAL)**

(Similar to Kreps, Problem 9, Chapter 17). In a particular population everyone runs the risk of losing $1000 randomly. Each person's chance to lose $1000 depends on the individual: fraction x of the population loses $1000 with probability .1 while the other fraction 1-x loses $1000 with probability 0.6. Individuals know their types, but insurance companies do not. Each individual is risk averse and has the same von Neumann-Morgenstern utility function given by $u(x) = -e^{-\lambda x}$ with $\lambda > 0$, but the insurance companies are risk neutral. Each individual decides whether to seek insurance or not. If he chooses to seek insurance, he approaches a number of insurance companies who simultaneously name premiums for full insurance P. The individual accepts the lowest premium. He pays the insurance company P. The company covers the $1000 loss in the event it happens.

A. What expected utility does an individual of each type gets if he chooses to go without insurance? What utility does he get if he chooses to take insurance?

B. Please think intuitively about whether the less risky individuals get insurance or not. How does the answer depend on λ and x (i.e. is there a separating equilibrium for large λ or small λ, large x or small x)? Why?

C. For what range of parameters x and λ is there a separating equilibrium? For what range of parameters is there a pooling equilibrium? Please verify your conjecture from part B.

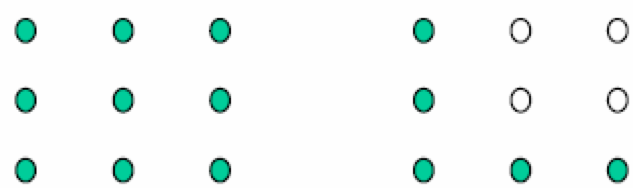
The Last Problem:

(Bonus) Consider the following two-player game. The "board" is an $m \times n$ grid of dots. Player 1 moves first, and chooses a dot. By choosing a particular dot, he removes this dot and all dots above and to the right of it, as illustrated in the picture. Player 2 moves second, and similarly chooses a dot, removing all dots above and to the right.

Then player 1 moves again, and so on. A player "wins" by forcing his opponent to remove the bottom left dot.

(a) Suppose that $m = n$, so the board is a square. Find a winning strategy for Player 1. (Hint: Start with the 2×2 case and work up).

(b) Prove that player 1 has a winning strategy in the general $m \times n$ game.



Initial 3x3 Board

Board after Center Square is removed.