Econ 201A. Part II.
Due: Tuesday, November 8

Problem Set 1

Homework policy: Please try to do the following problems on your own first. If you get stuck, feel free to discuss them with other people in the class, but acknowledge any discussion or ideas that you get on your homework, e.g. "I benefited from discussion with so-and-so on problem x." Please write your solutions clearly and concisely.

Problem 1. A two-player game with finitely many actions is played repeatedly. Imagine the following kind of dynamics:

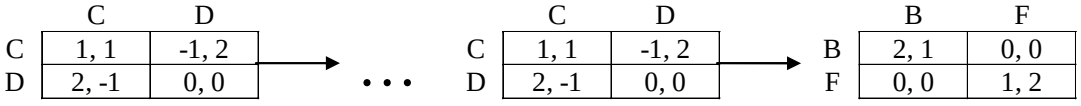
- In period 1, players choose a given pair of actions (a_1,a_2)
- In each *even* period thereafter, player 1 chooses the same action as in the previous period, and player 2 chooses a best response to that action (e.g. in period 2 player 2 chooses a best response to a_1).
- In each *odd* period, player 2 keeps the action from the previous period, while player 1 chooses a best response to that action

(a) **in class – no need to submit this part:** If the game has a unique pure strategy Nash equilibrium, will the players always converge to it? Prove, or give a counterexample.

(b) If the game has a unique pair of strategies that survives iterative elimination of strictly dominated strategies, will the players converge to it? Prove, or give a counterexample.

(c) Suppose that players never converge. Is there always a mixed strategy Nash equilibrium that involves *only* the actions used along the path of play? Prove or give a counterexample.

Problem 2: Prisoner's dilemma repeated *n* times and the Battle of the Sexes



Consider pure strategy SPE of the game above. Is it possible to have cooperation in the first period? What is the maximal number of periods that the players can cooperate? Specify a SPE in which the players cooperate for the maximal number of periods, that is, specify their actions in every single subgame.

Problem 3. Consider the following bargaining situation. Two individuals are considering undertaking a business venture that will earn them $100 in profit, but they must agree how to split the $100. Bargaining works as follows: The two individuals simultaneously make a demand. If their demands sum to more than $100, then they fail to agree and each gets nothing. If their demands sum to $100 or less, they do the project, each gets his demand, and the rest goes to charity (which neither values). What are the pure strategy Nash equilibria of this game?

Problem 4. Consider a game in which a particular player has N information sets, indexed by n = 1, 2, ...N. Suppose that he has M_n possible actions at information set n. How many strategies does he have in all?

Problem 5. Consider the following game in normal form, where player 1 chooses rows and player 2 chooses columns (in each cell, player 1's payoff is listed first and player 2's second).

	A	b	c	d	e
A	2, 1	4, 2	1, 0	10, 3	2, 4
B	6, 10	2, 20	0, 10	7, 15	0, 18
C	5, 0	0, 0	3, 3	5, 0	0, 1
D	8, 1	2, 4	0, 0	5, 1	4, 2
E	1, 3	2, 6	0, 4	8, 5	1, 3

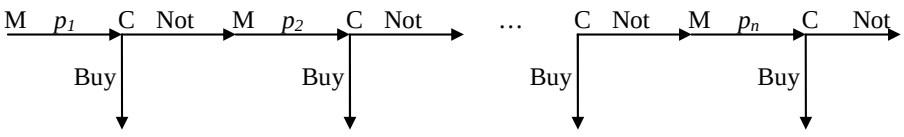
A. Does either player have strictly dominated strategies? If so, what are they?

B. What strategies are eliminated through iterative deletion of strictly dominated choices? Is the game dominance solvable?

C. Completely characterize the set of pure strategy Nash equilibria for this game.

D. Completely characterize the set of mixed strategy Nash equilibria for this game.

Bonus Problem: Consider the durable goods monopoly example from the lecture. There is a monopolist with zero marginal cost and potential customers whose valuations are uniformly distributed on the interval [0,1]. Suppose that there are n periods. The timing is represented as follows:



In each period *t*, the monopolist makes a price offer, and every customer who has not bought the good before decides whether to buy or not. If a buyer buys at time *t*, he leaves the market and gets utility $(n+1-t)v - p_t$. In other words, he pays p_t and enjoys the good

for ($n+1-t$) periods. We showed in class that in equilibrium there will be a decreasing sequence of valuations $1 = v_1 \geq v_2 \geq \ldots v_n$ such that in the beginning of period t customers in the interval $[0, v_t]$ are remaining. Please provide recursive formulas for v_t and equilibrium prices p_t. How does the monopolist's profit in this dynamic monopoly compare with his profit in a static monopoly (where the monopolist sets one price in period 1 for all periods).