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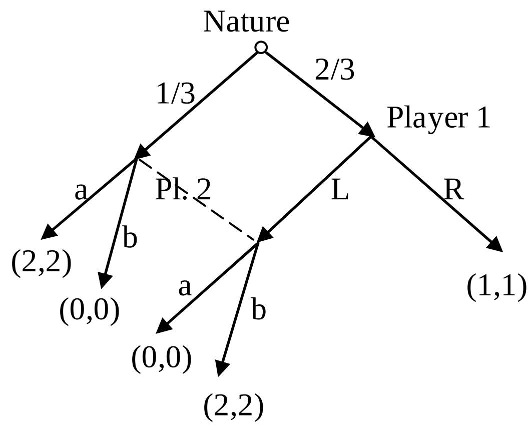
Economics 201A December 20, 2004
Game Theory

Final Exam Solutions

Problem 1. (20 points) Perfect Bayesian Equilibria:

	Player 1's strategy	Player 2's strategy	Player 2's belief
Equilibrium 1	L	b	(1/3, 2/3)
Equilibrium 2	R	a	(1, 0)
Equilibrium 3	1/2 L + 1/2 R	1/2 a + 1/2 b	(1/2, 1/2)



Problem 2. (25 points) There are 2 types with cost of education $c_L(e) = e^2$ and $c_H(e) = ae^2$, where $a \in (0,1)$. The productivities are $1 + e$ and $2 + e$ respectively.

(a) Type L must choose the efficient education level, one that maximizes $1 + e - c_L(e)$. The first order condition $c'_L(e) = 1$ gives $e = 1/2$.

(b) The equation for the indifference curve of type L that passes through point $(1/2, 1.5)$ is $1.25 + e^2$. It intersects the wage offer curve of type L where $1.25 + e^2 = 2 + e$, i.e. $e = 1.5$.

(c) The efficient level of education for type H solves $c'_H(e) = 1$, $e = 1/(2a)$.

(d) In the separating equilibrium that satisfies the intuitive criterion type H chooses the efficient level of education when $1/(2a) \geq 1.5$, i.e. $a \in (0, 1/3]$.

Problem 3. (25 points) There is a seller with one item and two potential buyers whose valuations are independently distributed according to a uniform distribution on $[0,1]$. An auction proceeds by the following rules. First, buyer 1 secretly submits a bid $b \geq 0$ to the seller. Upon seeing the bid of buyer 1, the seller can make any take it or leave it offer p to buyer 2. If buyer 2 accepts, the item goes to buyer 2 for the price of p. If buyer 2 rejects, the item goes to buyer 1 for the price of b (the amount of 1's bid).

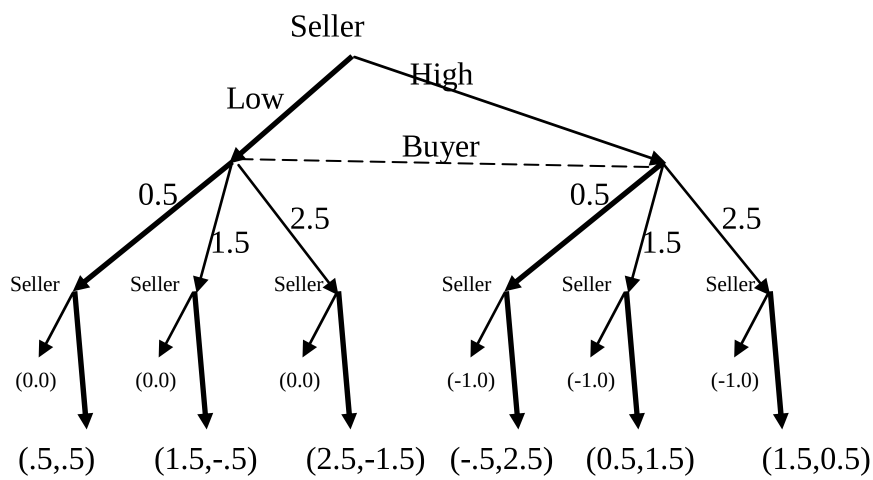
(a) If the seller offers price p to buyer 2, buyer 2 accepts the offer if his valuation falls in the range $[p, 1]$, which happens with probability $1 - p$. Then the seller's expected payoff function is $p(1-p) + bp$, which is maximized when the seller makes an offer of $p = \frac{1+b}{2}$.

(b) If buyer 1 bids b, he will win the auction with probability $(1+b)/2$. His expected payoff is $(v_1 - b)(1+b)/2$, which is maximized when $b = 0$.

(c) Given $b = 0$, the seller will offer $p = 1/2$ to buyer 2, and obtain an expected revenue of $p(1-p) + bp = 1/4$.

Problem 4. (30 points) Consider the following game. There is a seller and a buyer. First, the seller produces an item with quality either $q = 0$ or 1. It costs 0 to produce a low-quality item and 1 to produce a high quality item. A high-quality item has value $v(1) = 3$ to the buyer and a low-quality one has value $v(0) = 1$. The buyer, who cannot observe the quality of the product, can offer a price p from among the following three: 0.5, 1.5 or 2.5. The seller can accept or reject. If the seller accepts, his payoff is $p - q$ and the buyer's payoff is $v(q) - p$. If the seller rejects, his payoff is $-q$ and the buyer's is 0.

(a), (b) Here is the game tree and the SPE:



The equilibrium payoffs are (.5, .5).

Now suppose that this game is repeated infinitely many times with the same seller and a different potential buyer in each period. The new buyer in each period knows the quality of all previously sold items. If the item is not sold in a given period, it goes bad and the seller needs to produce a new item in the following period. The seller's discount factor is δ. Note that the repetition of a SPE from part (b) constitutes a SPE of the repeated game. This SPE is not ideal for the seller. Ideally, the seller would like to produce a product of high quality in every period and be offered a price of 2.5.

(c) Under these conditions if the seller always produces a high-quality good, he gets a price of 2.5 in every period and a total payoff of $\frac{2.5-1}{1-\delta}$. If he deviates once, he gets a price of 2.5 for a low-quality good in one period, and price 0.5 thereafter, with a total payoff of $2.5 + \frac{0.5\delta}{1-\delta}$. It is better not to deviate if $\frac{1}{1-\delta} \geq 2$, i.e. $\delta \geq 1/2$.

(d) In reality, the buyers can also offer a lower price and the seller must commit not to accept any price lower than 2.5. The most tempting deviation for the seller is to accept the price 1.5 if offered. If he accepts the price, he gains 1.5 currently, but loses $\delta/(1-\delta)$ in continuation value. The seller can commit not to accept the offer of 1.5 if $\delta \geq 3/5$.