

7. This mechanism is clearly not incentive compatible.

To see this, let us denote by $\{v_1, \dots, v_{N+1}\}$ the true valuations of the good for the $N+1$ consumers, where v_n represents the n th highest valuation. Clearly, the maximum amount that a consumer would be willing to pay, for the provision of the good, is equal to the value v that he derives from consumption of the good in question.

Consider the strategic decision problem of the i th player. Since there is no linkage between his bid and the amount he would have to pay if the good is provided, whatever his bid, he will end paying C if the good is actually provided. Hence, conditional on the provision proposal going through, bidder i will get an expected payoff of

$$(v_i - C) \Pr[v_{N/2} \geq C] \quad (I)$$

where $\Pr[v_{N/2} \geq C]$ denotes the probability that the $\frac{N}{2}$ -largest value amongst the voters is larger than C (and, therefore, the good gets to be provided).

Equation (I) shows that:

1. Bidders with values lower than C are clearly better off by the good not being provided. Nevertheless, if all other bidders are reporting their true values, the i th player with a true value lower than C has actually no incentive to announce a value different than his own. Announcing a value above his true value is clearly sub-optimal for him as it increases the probability of the good being provided (since it raises the number of people who are willing to pay for the good a price higher than C). Yet, announcing a value lower than his true value is not necessarily optimal either since, again, it does not change the number of people who are voting against¹.
2. Bidders with values higher than C are better off by the good being provided. Nevertheless, if all other bidders are reporting their true values, the i th player with a true value higher than C would have no incentive to announce a value higher than his true value since, he cannot alter the probability of the good being provided anyway. Announcing a value lower than your true one, is clearly not optimal as it decreases the number of voters with a price above C and, consequently, decreases the likelihood of the good being provided.

Note that our argument above establishes only that the mechanism is weakly incentive compatible. Given that everyone else is reporting their true values, reporting yours is an equilibrium. But it is by no means uniquely optimal as there is an infinite range of values that one can report in each of the cases above without altering his expected payoff.

¹ The interesting point of this setting is that one's bid is not linked to one's payoff once the good is been provided. Moreover, and in contrast to even a second-price auction, even the magnitude of the bid cannot really affect the probability of a given outcome (such as the good being provided). All that matters really when it comes to one's vote is if the maximum cost that they declare is higher or not than C . The extent of the discrepancy between it and C is irrelevant. This is exactly what is driving the result here and makes reporting one's true value an optimal strategy. Conditional on your true value being higher (lower) than C , reporting a value higher (lower) than your true value does not alter your expected payoff.