

**Structural Analysis of Discrete Data  
with Econometric Applications**

edited by  
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and Daniel McFadden

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## **Preface**

The chapters in this volume are all original and previously unpublished major research contributions made by econometricians to the structural analysis of discrete data. Two factors led to our decision to organize this volume.

First, we feel that the piecemeal publication in the journals of new research in this field has made it difficult for econometricians not actively working on discrete data problems to overview the existing state of knowledge and the present frontiers of research. Coordinated publication of the basic findings in this new subject should lower the cost of entry into the field and speed dissemination of recent research into the graduate econometrics classroom.

Second, as the econometric literature on discrete data analysis has grown, and its contributions have matured, we have increasingly wished to communicate the concerns and results of this literature to the wider community of researchers involved in the structural analysis of discrete data, both in applied economics and in disciplines outside economics. As the editors' introduction to this volume emphasizes, there exist important interconnections between the econometric literature on discrete data and the work in statistics, biometrics, psychometrics, sociometrics, and other disciplines on discrete data analysis. We have tried to organize this volume so that readers from outside economics as well as applied economists will recognize the connections between the problems they face and the issues addressed in econometric analysis of discrete data and will find the results obtained here useful in their work.

Submissions of papers for possible publication in the volume were solicited by the editors in the spring of 1978. We are grateful to R. Avery, M. Ben-Akiva, S. Cosslett, G. Duncan, D. Gillen, J. Hausman, J. Heckman, L. Lee, S. Lerman, D. Nagin, D. Poirier, P. Schmidt, B. Singer, R. Westin, and D. Wise, each of whom reviewed one or more papers.

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