13 Estimating Credit Constraints by Switching Regressions

Robert B. Avery

13.1 Introduction

Congress recently passed the Equal Credit Opportunity Act, ECOA, which prohibits firms from discriminating on the basis of race, sex, or age in the granting of credit. The ultimate impact of ECOA cannot be determined until precise definitions of discrimination are decided by the judicial system. It is clear, however, that the effect of ECOA will depend largely upon the relationship between race, sex, age, and credit risk, hence credit availability. This essentially empirical issue has not yet been satisfactorily addressed. It is the intent of this chapter to investigate empirically the pre-ECOA relationship between credit availability and variables cited in the act, using a methodology that avoids many of the problems of earlier studies. Although ECOA covers all credit markets and the variables cited, we shall focus on the consumer credit market and race. We define consumer credit to include virtually all short-term, nonmortgage household borrowing. Although other types of borrowing are common, this type of lending is generally associated with durable goods purchases. Consumers are likely to owe small amounts to a number of lenders scattered among banks, stores, credit unions, and consumer finance companies.

Empirical evidence from previous studies linking race and the availability of consumer credit appears to be contradictory. Shinkel (1976), in the only comprehensive study on the effects of ECOA, examines a stratified random sample of approximately 10,000 national consumer finance company loan customers for evidence of racial effects in credit riskiness. Shinkel divides his sample into good loans (those repaid) and bad loans (those defaulted). He examines differences between the two samples with respect to a set of variables generally available at the time of the loan application, using multiple discriminate analysis. He concludes that one can discriminate between the two samples almost as well with an information set excluding race as when race is included. Excluding race increased the bad loans accepted by 0.5 percent and reduced the good loans accepted by 2.3 percent, resulting in a drop of at most 7 percent in firm profits.

Shinkel presents some other interesting evidence. He cites a survey of 100 credit-scoring systems conducted by Fair, Isaac and Company that showed neither race nor color was used by any firm. This does not necessarily imply that race is not, or never has been, used in credit screening, as many firms rely on subjective judgment even if they also use credit-scoring systems. This author conducted interviews with a number of New Jersey consumer loan officers, with the conclusion that race indeed was used. Most loan officers indicated, however, that race was not a major factor and was useful mainly in screening low-income applicants.

In contrast to the evidence that race does not play a large role in firm credit screening procedures, is the evidence that blacks, particularly in low-income classes, are observed to hold substantially lower amounts of consumer debt than comparable whites. Bell (1974) using the 1967 Survey of Economic Opportunity, SEO, data file concludes that black family renters with income less than $3,000 hold between 35 and 79 percent of the debt of comparable whites, depending on geographic location. Families with income between $3,000 and $6,000 hold between 59 and 73 percent of the debt of comparable whites. Bell also concludes that young black families in particular are likely to hold substantially less consumer debt than comparable whites.

Is the evidence that at the firm level race appears to only a small factor in the granting of credit inconsistent with evidence that in the aggregate blacks in fact receive less credit? One possible explanation is that blacks demand less credit. Kain and Quigley (1972), however, present empirical evidence that blacks pay higher per unit prices in the housing market than comparable whites. They argue that since blacks face higher housing prices, their demand for substitute goods, such as automobiles and durables may increase, altering their composition of wealth. Since durables are generally associated with short-term debt, blacks may demand higher not lower levels of consumer debt than comparable whites.

We give an alternative explanation in Avery (1977). We argue that Shinkel is measuring a firm-based decision rule that critically depends on who applies for loans. There is a cost to applicants in making loan applications, however. Those applicants with high probabilities of loan denials may not bother applying. Similarly firms may effectively screen large segments of the black community by not locating offices in black neighborhoods. We demonstrate that self-selection on the part of applicants may produce firm decision rules that differ substantially in their treatment of race from rules that would be used if all potential applicants were to apply. This suggests that Bell is in effect measuring a different concept than Shinkel. The amount of credit that ultimately flows to blacks is a function not only of explicit firm credit screening procedures and consumer credit demand but indirect credit screening via office location or induced applicant self-selection.

There is a grave danger that ECOA will only affect explicit credit screening by race and ignore the potentially more serious form of indirect screening. It is the interest of this chapter to examine and measure the latter concept. We propose to do this via a household not firm-based model. As we argued earlier, firm-based empirical models cannot be used to measure indirect screening. It would also be insufficient, however, to use household data and simply regress aggregate observed levels of debt against a series of demographic variables, isolating the sign and magnitude of a coefficient on race. The observed quantity of debt is as much a function of the demand for debt as the supply. To the extent that the demand for debt is correlated with race, the coefficient on race in such a reduced form regression would be a mixture of supply and demand effects.

As an alternative we propose to estimate a model of household behavior that combines both a demand and supply function for consumer debt. Since we believe that both the supply and demand for debt are functions of simultaneously determined household decisions, we also propose to incorporate behavioral equations thought to be closely related to consumer debt. Our broad interest is in the estimation of a consumer debt supply function, and specifically the role of race.

13.2 The Supply of Debt

The supply of consumer debt is a concept that may need further elaboration. The concept of a consumer loan supply function or credit rationing is not original with this study. Friedman (1957), Tobin (1957, 1972), Tobin and Watts (1960), Dolde and Tobin (1971), Watkins (1975), and Anderson (1976), each allude to the existence of credit constraints within the context of household demand models. None of these authors, however, specified a credit supply function, nor did they incorporate such constraints in estimation. Harris (1974) and Peterson (1976) explored aggregate credit supply functions, but neither author considers such a function at the micro level.

At the heart of the concept of a liquidity constraint or credit supply function is the view that credit is rationed, that firms fix the nominal rate of interest and deny loans to all applicants with expected net returns below a predetermined threshold. Thus all variation in applicant rates of return stems from their ability to repay the loan, not differential interest rates. One explanation for fixed interest rates is state usary laws that impose binding ceilings on consumer credit interest rates in most states. Modigliani and Jaffee (1970) also argue that rationing may arise endogenously. They argue that due to market imperfections and transactions costs it is optimal for American financial institutions to segment the lending market into a small number of segments and within each segment charge the same interest rate.

We argued in another study (Avery 1977) that faced with fixed interest rates rational profit-maximizing firms will collect data from prospective applicants. They will use this information to compute an expected return, granting loans where the expected return exceeds a predetermined threshold. This process, which we believe accurately describes much of the consumer credit market, is a loan-by-loan process. If, however, an applicant's existing stock of debt is one of the variables used in screening loans, it can easily be shown that the process reduces to a credit limit or maximum amount of debt for each applicant. Furthermore, this credit limit will be a function of other variables used in credit screening. To the extent that all firms use similar screening devices, the process yields a market credit limit.

We admit that it may be unrealistic to assume that consumers face one market credit limit. Even if interest rates are fixed within segments of the market, consumers have access to other sources of credit, for example, loan sharks, at higher interest rates. However, we see no practical way of addressing this problem, particularly given the limitations of our data. We are forced to assume that price variations are minor and that consumers face one consumer credit limit.

Finally, we argue that firms screen credit applicants by both direct and indirect means. As we argued earlier, firms may be able to indirectly screen applicants by office location. Similarly applicants may anticipate firm-screening procedures and not bother applying for loans that are likely to be turned down. Thus the implicit credit-screening procedure may differ from the process used by firms to evaluate actual credit applicants. We are interested in estimating the total credit supply function or credit limit, recognizing that it is achieved via a combination of both direct and indirect screening.

13.3 The Model and Data

We propose a cross-sectional stock demand model similar in spirit to that of Tobin (1957) and Watts and Tobin (1960). At a given point in time households are assumed to maintain equilibrium stocks of debt and durables. Household behavior is modified by wealth, income, and debt constraints. Desired household durable holdings are assumed to be functions of demographic characteristics and debt holdings and a measure of life cycle or permanent income, with stochastically distributed error terms accounting for omitted variables such as personality. Each of these is taken as exogenously given and independent of portfolio decisions. Households are assumed to be subject to debt or liquidity constraints. The maximum amount of debt allowed a household is assumed to be a function of demographic variables, durable stock holdings, income, credit history, plus errors accounting for firm discretion or omitted variables. Households are also assumed to have a well-specified demand for debt, which is a function of durable stock holdings and the same exogenous variables that affect durable demand. The actual observed household debt holding will be the minimum of debt demand and supply. Since our empirical model is to some extent shaped by the characteristics of our data, before outlining the specific model, let us briefly describe the data base.

Data were drawn from the New Jersey negative income tax experiment. The experiment was conducted primarily to measure the labor supply response of low-income families to a negative income tax system. Roughly 1,300 low income families were selected in four New Jersey cities—Trenton, Jersey City, Paterson, and Passaic—and in Scranton, Pennsylvania. Families were then assigned to a control-sample, or to one of a set of negative income tax plans. The negative income tax plans were described by a guarantee—an amount paid the family whether they work or not—and a tax rate, or the fraction of a dollar that negative income tax payments are reduced for each dollar of family earnings. Families were allocated to one of eight experimental plans characterized by guarantees ranging from 50 to 125 percent of the poverty level and tax rates of 0.3, 0.5, or 0.7.

The experimental data represents a stratified sample of low-income families. This, however, offers a particularly good sample for our problem. Bell (1974) argues that black-white credit differentials were most pronounced in low-income groups. It can also be argued that low-income families are the most likely to be debt constrained. Thus such a sample offers more information on the equation of particular interest, credit supply.

The experiment ran for three years in each city, starting in Trenton in 1968 and ending in Scranton in 1972. Participants were asked a series of questions prior to enrollment and approximately every three months thereafter. Roughly half the original participants either dropped out of the experiment or had major gaps of missing data, leaving a sample of 604 for whom complete data are available for the three-year period.

Although the New Jersey experiment data base is in panel form, 13 drawings over time on each family, we shall treat it as cross-sectional. Several of the critical debt variables are available only once, near the end of the experiment. For this reason it was decided to view the sample as cross-sectional and to utilize the temporal nature of the data in constructing variables rather than directly. The specific variables used are listed in table 13.1 with sample means and standard deviations. Several of the variables, such as age and asset stocks, show considerable variation over the course of the experiment. Each of these variables therefore was taken as of the tenth quarter of the experiment when the richest set of financial variables were available. Most of the independent variables are self-explanatory.

Thirteen quarterly drawings on weekly family income and earnings were available over the course of the experiment. These were averaged to provide a measure of stable income. In addition the variance of income was computed about the three-year average. Total income comprises income from all sources, including welfare and experimental payments, other unearned income, and all family earnings.

The negative income tax experiment coincided with the 1970 census, affording access to some interesting neighborhood variables. The 604 sample families were located in roughly 100 different census tracts. Data from these census tracts were used to compute the neighborhood variables.

We choose to account for the effects of the experiment itself in two ways: first, by a 0–1 dummy variable, allowing a mean shift for the 60 percent of the sample families on one of the eight experimental plans, and second, by including experimental payments in the income variables.

Construction of the endogenous variables was a little more complex and needs further explanation. We discuss this within the context of our specific model equations.

Table 13.1
List of variables used

	Symbol	Sample mean	Sample standard deviation
Endogenous variables			
Value of automobiles stock (100's \$)	A	4.7	8.4
Value of other durables (100's \$)	O	9.9	5.8
Stock demand for consumer debt (100's \$)	D	—	—
Stock supply of consumer debt (100's \$)	S	—	—
Actual quantity of debt (100's \$)	Q	8.5	10.7
Independent variables			
Age of family head			
≤ 25 years	A_1	0.04	0.19
26–35 years	A_2	0.33	0.47
36–45 years	A_3	0.37	0.48
46–55 years	A_4	0.21	0.41
$56 \leq$ years	A_5	0.05	0.23
Education of family head			
< 8 years	E_1	0.27	0.44
8–11 years	E_2	0.50	0.50
$12 \leq$ years	E_3	0.23	0.42
Family demographics			
total number of persons in family	F_1	6.3	2.2
number of adults	F_2	2.6	0.9
Health			
dummy bad health family head	HL_1	0.30	0.34
dummy bad health spouse	HL_2	0.30	0.32
Homeownership dummy	H	0.23	0.42
Family income			
average total weekly income (from all sources) over three year period (\$)	I_1	141.0	41.5
average weekly head earnings (\$)	I_2	95.7	39.3
average weekly spouse earnings (\$)	I_3	7.5	18.0
ratio of average earned to total income	I_4	0.78	0.21
variance of earnings over three year period (\$)	I_5	34.1	20.2
dummy if total average income > \$100 a week	I_6	0.85	0.35
dummy if ever on welfare during three-year period	I_7	0.34	0.48

Table 13.1
(continued)

	Symbol	Sample mean	Sample standard deviation
Neighborhood characteristics (by census tract)			
median weekly family income ($)	N_1	146.3	23.9
dummy if neighborhood median income above poverty level	N_2	0.72	0.45
fraction of neighborhood that is black	N_3	0.28	0.31
ratio of median neighborhood housing value to income	N_4	122.5	39.1
Negative income tax experimental family	X	0.62	0.49
Race			
white	W	0.44	0.50
black	B	0.33	0.47
Spanish-speaking	SP	0.23	0.42
Credit supply function variables			
average tenure at last two jobs for head (years)	C_1	6.0	5.1
dummy if lived in residence less than one year	C_2	0.38	0.49
dummy if ever repossessed or wages garnished	C_3	0.07	0.26

Auto and Durable Stock Demand

Although there are many household asset decisions, we choose to model only those thought to be closely related to consumer debt decisions. In particular we model only durable stock decisions, which we believe are generally made simultaneously with consumer debt. Because we think they are related differently to consumer debt, particularly credit supply, we separate durables into two classes: (1) automobiles and (2) all other durables. We assume that stock holdings of autos A and other durables O will be linear functions of the same set of exogenous demographic variables, the quantity of consumer debt Q and ε_1, ε_2 stochastic normal errors. Formally

$$A = f_A(Q, A_2, A_3, A_4, A_5, E_2, E_3, F_1, F_2, HL_1, HL_2, H, I_1, I_4, I_5, N_1, N_3, N_4, X, B, SP, \text{constant}, \varepsilon_1); \quad (13.1)$$

$$O = f_O(Q, A_2, A_3, A_4, A_5, E_2, E_3, F_1, F_2, HL_1, HL_2, H, I_1, I_4, I_5, N_1, N_3, N_4, X, B, SP, \text{constant}, \varepsilon_2). \quad (13.2)$$

The stock value of automobiles was estimated as a function of the age and make of the car. Other durables include both appliances and furniture. Stock values were computed from purchase price, where available, or by extrapolation. A detailed description of the methods used to calculate stock valuations of both automobiles and other durables is available in Metcalf (1977).

Debt Demand

Households are assumed to have a stock demand for consumer debt that is a function of the same exogenous variables as autos and other durables and to be simultaneously determined with these variables. Defining consumer debt as the dollar sum of all nonmortgage household debt, we assume

$$D = f_D(A, O, A_2, A_3, A_4, A_5, E_2, E_3, F_1, F_2, HL_1, HL_2, H, I_1, I_4, I_5, N_1, N_3, N_4, X, B, SP, \text{constant}, \varepsilon_3), \quad (13.3)$$

where D is debt demand and again ε_3 is a stochastic normal error.

The assumption that debt, durable, and auto demand are functions of the same variables will be shown later to imply that the parameters of the debt demand equation are not fully identified. We will ultimately estimate a reduced form where debt demand is a function only of the exogenous variables. Thus the effects of A and O will be indistinguishable from those of the exogenous variables.

Debt Supply

Debt supply is the equation of primary interest to our study and the only one resembling a structural or behavioral equation. Consumers are assumed to face a market credit limit that varies from household to household. Firms are assumed to be able to constrain households at a fixed credit price by a combination of direct and indirect screening. Variables for the supply function were selected on the basis of interviews with a number of loan officers drawn from the experimental area. Exogenous variables mentioned most frequently as being used in credit screening were income, domestic and job stability, demographics, and credit history. We have tried to include measures of these variables similar to those seen on loan application forms. Formally we assume that the supply of debt is a linear function,

$$S = f_S(A, O, A_1, A_5, F_1, H, I_2, I_3, I_6, I_7, N_2, B, SP, C_1, C_2, C_3, \text{constant}, \varepsilon_4), \tag{13.4}$$

where ε_4 is again a normally distributed error.

We have assumed debt supply to be a function of auto and durable stocks. The inclusion of these variables dictates our particular model specification. We could have specified both credit supply and demand as reduced forms, functions only of exogenous variables. This would have made the model much simpler and avoided the necessity of estimating durable and debt equations simultaneously. However, we are interested in the effects of race on credit supply, holding other variables constant. To the extent that important variables like durable holdings may vary by race, they must be controlled for explicitly in the credit supply equation. Since we believe that durable decisions are often made simultaneously with debt decisions, this also requires us to specify the linkages between debt and durables in the durable equations we have noted.

Marketing Clearing

In most economic models we assume market clearing, supply equals demand, as an equilibrium condition. Our earlier discussions of the consumer credit market suggest, however, that this is an inappropriate condition for supply and demand as we have defined it. We argued in our discussion of debt demand that given a fixed interest rate, firms would establish a credit limit for each household. Thus credit supply is really an upper bound or constraint. Viewed this way, there is no reason to believe that supply will equal demand. This will affect the relationship between debt demand and supply and the actual amount of debt observed. If a household's debt demand were less than its market credit limit, observed debt would equal that demanded and the constraint would be irrelevant. However, if demand were to exceed the credit limit, the constraint would be binding, and we would observe actual debt equal to debt supply. This implies that

$$Q = \min(S, D), \tag{13.5}$$

where Q is the observed quantity of consumer debt. Note that this specification implies that we will observe either debt demand or debt supply for a given observation, never both.

There are a number of potential weaknesses of our model which bear some comment. First, some of the variables we call exogenous, such as income and housing stocks, cannot be said to be truly independent of portfolio decisions. Cain (1967) and Mincer (1960), for example, argue persuasively that earnings, particularly of secondary workers, may be sensitive to asset holdings, particularly constraints on debt. Housing stocks, particularly due to their dependence on mortgage debt (which is not included in consumer debt), are also likely to be influenced by consumer debt stocks. Second, households are assumed to be price-takers and to face the same prices for durables and debt. The assumption of a fixed debt price is particularly critical. To the extent that it is violated, demographic variables may pick up some price effects. Third, all households are assumed to be in equilibrium. This is perhaps the least defensible of our assumptions. Clearly at any point in time some households will be out of equilibrium. Their income may have suddenly fallen, yet there may not have been sufficient time to adjust their portfolio. We are forced to assume, however, either that all households are in equilibrium or that deviations from desired stocks are independent of the exogenous variables. Finally, and most important, is the reduced form flavor of our model. Our demand equations lack the behavioral characteristics normally found in structural economic models. To the extent that these reduced forms so not adequately explain observed variations in demand, the whole model may suffer.

The Complete Model System

The model system can be represented compactly as follows:

Autos: $$A = \mathbf{X}_1\boldsymbol{\beta}_1 + \alpha_{11}Q + \varepsilon_1 \tag{13.6}$$

Other durables: $$O = \mathbf{X}_1\boldsymbol{\beta}_2 + \alpha_{21}Q + \varepsilon_2 \tag{13.7}$$

Demand for debt: $$D = \mathbf{X}_1\boldsymbol{\beta}_3 + \alpha_{31}A + \alpha_{32}O + \varepsilon_3 \tag{13.8}$$

Supply of debt: $$S = \mathbf{X}_2\boldsymbol{\beta}_4 + \alpha_{41}A + \alpha_{42}O + \varepsilon_4 \tag{13.9}$$

Observed debt: $$Q = \min(S, D) \tag{13.10}$$

with $\mathbf{X}_1$ and $\mathbf{X}_2$ representing different exogenous variable sets. The fact that S and D are unobserved, coupled with the simultaneous determination of the five equations, creates some difficulties in estimation. In section 13.4 we address the issue of estimation and derive a procedure to compute consistent parameter estimates. Actual estimates of the model are presented in section 13.5.

13.4 Simultaneous Switching Regression and Linear Equations

The model system described in the previous section consists of two linear equations and a switching regression. For illustrative purposes consider the smaller system containing only one linear equation:

$$Y_n = \mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1 Q_n + \varepsilon_{1n}, \tag{13.11}$$

$$D_n = \mathbf{X}'_{2n}\boldsymbol{\beta}_2 + \alpha_2 Y_n + \varepsilon_{2n}, \tag{13.12}$$

$$S_n = \mathbf{X}'_{3n}\boldsymbol{\beta}_3 + \alpha_3 Y_n + \varepsilon_{3n}, \tag{13.13}$$

$$Q_n = \min(D_n, S_n), \tag{13.14}$$

$n = 1, \ldots, N$, where $\mathbf{X}_{1n}$, $\mathbf{X}_{2n}$, and $\mathbf{X}_{3n}$ are K_1, K_2, and K_3 length column vectors of independent variables, and ε_{1n}, ε_{2n}, ε_{3n} are stochastic errors, serially independent, and contemporaneously distributed $N(\mathbf{0}, \boldsymbol{\Sigma})$.

This system can be interpreted as the simultaneous solution of two demand equations, D and Y, complicated by an unobserved constraint on the value of D, S. We could also view D and S as the unobserved quantity demanded and supplied of variable Q, with only their minimum observed. The latter view is identical to the interpretation given the model presented in section 13.3.

Taken by themselves, equations (13.12), (13.13), and (13.14) resemble a switching regression or markets in disequilibrium model first introduced by Fair and Jaffee (1972) and later modified by Amemiya (1974) and Maddala and Nelson (1974). Were it not for the simultaneity, Hartley and Mallela (1977) show that strongly consistent parameter estimates for (13.14) and (13.13) could easily be derived from maximum likelihood techniques under very general conditions. However, the simultaneous determination of Q and Y necessitates the use of a more complex estimation procedure. In the remainder of this section we outline a procedure that can be used to compute consistent estimates of the parameters of (13.11) through (13.14). We start by deriving the system's reduced forms.

There are only two possible equilibrium reduced forms: either observed debt is equal to supply, or it is equal to demand. We solve (13.11) through (13.14) under each of these assumptions. Assume for the moment that Q is supply constrained, $Q_n = S_n$. Solving (13.11) and (13.13) simultaneously yields the reduced forms

$$Y_n = Y_n^s = \frac{\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}'_{3n}\boldsymbol{\beta}_3}{1 - \alpha_1\alpha_3} + \frac{\alpha_1\varepsilon_{3n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_3}, \tag{13.15}$$

$$Q_n = S_n^s = \frac{\mathbf{X}'_{3n}\boldsymbol{\beta}_3 + \alpha_3\mathbf{X}'_{1n}\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_3} + \frac{\alpha_3\varepsilon_{1n} + \varepsilon_{3n}}{1 - \alpha_1\alpha_3}, \tag{13.16}$$

which we denote with superscript s. Furthermore we can solve for unobserved demand D_n^s as

$$D_n^s = \mathbf{X}'_{2n}\boldsymbol{\beta}_2 + \frac{\alpha_2\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\alpha_2\mathbf{X}'_{3n}\boldsymbol{\beta}_n}{1 - \alpha_1\alpha_3} + \frac{\alpha_2\varepsilon_{1n} + \alpha_1\alpha_2\varepsilon_{3n}}{1 - \alpha_1\alpha_3} + \varepsilon_{2n}. \tag{13.17}$$

Conversely suppose that the system is demand constrained, $Q_n = D_n$. Solving (13.11) and (13.12) yields the reduced forms

$$Y_n = Y_n^d = \frac{\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}'_{2n}\boldsymbol{\beta}_2}{1 - \alpha_1\alpha_2} + \frac{\alpha_1\varepsilon_{2n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_2}, \tag{13.18}$$

$$Q_n = D_n^d = \frac{\mathbf{X}'_{2n}\boldsymbol{\beta}_2 + \alpha_2\mathbf{X}'_{1n}\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_2} + \frac{\alpha_2\varepsilon_{1n} + \varepsilon_{2n}}{1 - \alpha_1\alpha_2}, \tag{13.19}$$

which we denote with superscript d. Unobserved supply S_n^d can be solved for as

$$S_n^d = \mathbf{X}'_{3n}\boldsymbol{\beta}_3 + \frac{\alpha_3\mathbf{X}_{1n}\boldsymbol{\beta}_1 + \alpha_1\alpha_3\mathbf{X}_{2n}\boldsymbol{\beta}_2}{1 - \alpha_1\alpha_2} + \frac{\alpha_3\varepsilon_{1n} + \alpha_1\alpha_3\varepsilon_{2n}}{1 - \alpha_1\alpha_2} + \varepsilon_{3n}. \tag{13.20}$$

Thus the reduced form solutions imply two possible reduced form systems, (13.15) through (13.17) which hold when $Q_n = S_n$, and (13.18) through (13.20) which hold when $Q_n = D_n$.

A yet unanswered question is whether a unique equilibrium will exist for all observations: Can the demand-constrained and supply-constrained regimes occur simultaneously, or can there be regions of the error space for which neither occurs? In the simple case where $S_n = D_n = Q_n$, the two reduced forms are identical, and the system will have a unique equilibrium. However, if S_n differs from D_n, we require restrictions on the parameters for the existence of a unique equilibrium. In particular we require that $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$ be the same sign. If and only if this condition is met, the system will have a unique equilibrium with either the demand-constrained regime or the supply-constrained regime prevailing for every observation. A proof of this assertion is omitted, because it is a straightforward algebraic exercise.

Assuming that conditions for a unique equilibrium are met, it should be possible to compute consistent parameter estimates of the two reduced form regimes simultaneously using a full information maximum likelihood. For computational simplicity and compatibility with existing computer algorithms, however, it may be desirable to consider a limited information procedure.

We have asserted that, given conditions for a unique equilibrium, we will observe either $Q_n = S_n^s$ or $Q_n = D_n^d$. We have not, however, addressed the issue of how the prevailing reduced form regime is determined. It can be shown that the determination of the prevailing reduced form depends on the sign of $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$. If both terms are positive, the supply-constrained regime will prevail when $S_n^s < D_n^d$ and the demand-constrained regime when $D_n^d < S_n^s$. Thus we will observe Q_n equal to the minimum of D_n^d and S_n^s. If $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$ are both negative, the observed Q_n will be equal to the maximum of D_n^d and S_n^s.

The observed value of Y_n is determined in a similar fashion, depending on the sign of α_1. If $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$ are positive, and if α_1 is also positive, we will observe Y_n equal to the minimum of Y_n^s and Y_n^d. If α_1 is negative, we will observe the maximum of Y_n^s and Y_n^d. If $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$ are negative, then these relationships will be reversed.

Assuming, as we shall throughout the remainder of this chapter, that $1 - \alpha_1\alpha_2$ and $1 - \alpha_1\alpha_3$ are both positive, we can thus represent the reduced forms as two switching regression models:

$$Y_n^s = \frac{\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}'_{3n}\boldsymbol{\beta}_3}{1 - \alpha_1\alpha_3} + \frac{\alpha_1\varepsilon_{3n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_3}, \tag{13.21}$$

$$Y_n^d = \frac{\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}'_{2n}\boldsymbol{\beta}_2}{1 - \alpha_1\alpha_2} + \frac{\alpha_1\varepsilon_{2n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_2}, \tag{13.22}$$

$$\begin{aligned} Y_n &= \min(Y_n^s, Y_n^d) \quad \text{if } \alpha_1 > 0, \\ Y_n &= \max(Y_n^s, Y_n^d) \quad \text{if } \alpha_1 < 0; \end{aligned} \tag{13.23}$$

and

$$S_n^s = \frac{\mathbf{X}'_{3n}\boldsymbol{\beta}_3 + \alpha_3\mathbf{X}'_{1n}\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_3} + \frac{\alpha_3\varepsilon_{1n} + \varepsilon_{3n}}{1 - \alpha_1\alpha_3}, \tag{13.24}$$

$$D_n^d = \frac{\mathbf{X}'_{2n}\boldsymbol{\beta}_2 + \alpha_2\mathbf{X}'_{1n}\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_2} + \frac{\alpha_2\varepsilon_{1n} + \varepsilon_{2n}}{1 - \alpha_1\alpha_2}, \tag{13.25}$$

$$Q_n = \min(D_n^d, S_n^s). \tag{13.26}$$

Hartley and Mallela (1977) show that under very general conditions MLE's of standard switching regression models will be strongly consistent. Since the reduced form equations otherwise meet the assumptions of the standard model, if they satisfy the general regularity conditions, then the ML methods can be applied to each reduced form separately. We require, however, knowledge of the sign of α_1. These estimates are likely to be less efficient than full information ML, however, since we are ignoring any cross-equation restrictions and that both reduced form systems switch at the same time. Note, even if the original structural equation errors are uncorrelated, the reduced form errors will be correlated as ε_1 enters each term.

Assuming conditions for identification are met, there may be a variety of methods that could be used to solve for structural parameters from the reduced forms. We propose a solution procedure that is computationally feasible and will produce consistent estimators. We describe our procedure and prove its properties as follows:

The reduced form equations (13.21) through (13.26) can be rewritten as

$$Y_n^s = \mathbf{X}_{1n}'\boldsymbol{\beta}_1 + \alpha_1 \bar{S}_n^s + \frac{\alpha_1\varepsilon_{3n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_3}, \tag{13.27}$$

$$Y_n^d = \mathbf{X}_{1n}'\boldsymbol{\beta}_1 + \alpha_1 \bar{D}_n^d + \frac{\alpha_1\varepsilon_{2n} + \varepsilon_{1n}}{1 - \alpha_1\alpha_2}, \tag{13.28}$$

$$D_n^d = \mathbf{X}_{2n}'\boldsymbol{\beta}_2 + \alpha_2 \bar{Y}_n^d + \frac{\alpha_2\varepsilon_{1n} + \varepsilon_{2n}}{1 - \alpha_1\alpha_2}, \tag{13.29}$$

$$S_n^s = \mathbf{X}_{3n}'\boldsymbol{\beta}_3 + \alpha_3 \bar{Y}_n^s + \frac{\alpha_3\varepsilon_{1n} + \varepsilon_{3n}}{1 - \alpha_1\alpha_3}, \tag{13.30}$$

where

$$\bar{S}_n^s = \frac{\mathbf{X}_{3n}'\boldsymbol{\beta}_3 + \alpha_3\mathbf{X}_{1n}'\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_3},$$

$$\bar{D}_n^d = \frac{\mathbf{X}_{2n}'\boldsymbol{\beta}_2 + \alpha_2\mathbf{X}_{1n}'\boldsymbol{\beta}_1}{1 - \alpha_1\alpha_2},$$

$$\bar{Y}_n^s = \frac{\mathbf{X}_{1n}'\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}_{3n}'\boldsymbol{\beta}_3}{1 - \alpha_1\alpha_3},$$

$$\bar{Y}_n^d = \frac{\mathbf{X}'_{1n}\boldsymbol{\beta}_1 + \alpha_1\mathbf{X}'_{2n}\boldsymbol{\beta}_2}{1 - \alpha_1\alpha_2}, \tag{13.31}$$

are the nonerror right-hand terms of the four equations (13.24), (13.25), (13.21), and (13.22), respectively.

Suppose for a moment that we could observe $\{\bar{S}_n^s, \bar{D}_n^d, \bar{Y}_n^s, \bar{Y}_n^d\}$. Assuming that the reduced forms (13.27) through (13.30) each satisfy the assumptions of a standard switching regression model, we could apply ML procedures to compute consistent estimates for the parameter set $\{\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3\}$. We propose an estimation procedure similar to this except that instruments are used for the unobserved variables. Specifically we propose using ML estimates of the two reduced forms (13.21) through (13.26) to obtain instruments for $\{\bar{S}_n^s, \bar{D}_n^d, \bar{Y}_n^s, \bar{Y}_n^d\}$. The four instruments are then plugged into (13.27) through (13.30) and quasi-MLE's of $\{\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3\}$ computed by maximizing the likelihood of (13.27) through (13.30) given the instruments rather than the true values $\{\bar{S}_n^s, \bar{D}_n^d, \bar{Y}_n^s, \bar{Y}_n^d\}$. We assert that if the model satisfies several very general conditions that our proposed estimators will be strongly consistent for $\{\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3\}$. We prove this assertion as follows:

We need to assume that each of the reduced forms (13.21) through (13.26) and (13.27) through (13.30) individually satisfies the conditions required by Hartley-Mallela (1977) for strong consistency.

Defining $\mathbf{X}_s$ and $\mathbf{X}_d$ as the exogenous variables, ε_s and ε_d as the errors, and $\boldsymbol{\theta}_s$ and $\boldsymbol{\theta}_d$ as the unknown parameters of the supply and demand equations, respectively, these conditions require that

1. the errors are bivariate normal, have zero mean, serially independent, and independent of $\mathbf{X}_s$ and $\mathbf{X}_d$,

2. the empirical distribution of $\{\mathbf{X}_s, \mathbf{X}_d\}$ converges completely to a nondegenerate distribution function and the empirical second moment of $\{\mathbf{X}_s, \mathbf{X}_d\}$ divided by N converges to a positive definite matrix $\mathbf{M}$,

3. the parameter space $\boldsymbol{\Theta}$ is compact where $\{\boldsymbol{\theta}_s, \boldsymbol{\theta}_d\}$ is an interior point, with positive error variances and error correlation less in absolute value than one,

4. the components of $\{\boldsymbol{\theta}_s, \boldsymbol{\theta}_d\}$ are functionally independent and $\mathbf{X}_s$ or $\mathbf{X}_d$ contain at least one variable specific to it.

Assumptions 13.1 and 13.3. follow from the original specification of the model. Assumption 13.2 restricts the process generating $\mathbf{X}_s$ and $\mathbf{X}_d$. It does not require, however, that $\mathbf{X}_s$ and $\mathbf{X}_d$ be random. Assumption 13.4, when applied to (13.21) through (13.26) requires that $\mathbf{X}_2$ or $\mathbf{X}_3$ have at least one variable unique to it. When applied to (13.27) through (13.30), it requires that the structural equation parameters be identified.

Define $\hat{\pi}_Y$ and $\hat{\pi}_Q$ as the estimators that maximize the sample likelihood of the reduced forms (13.21) through (13.23) and (13.24) through (13.26), respectively. Let π_Y and π_Q be the true parameters of these equations. Furthermore let

$$L_Y(Y, \pi_Q, \boldsymbol{\theta}_Y)$$
$$L_Q(Q, \pi_Y, \boldsymbol{\theta}_Q)$$

be the sample likelihood of the structural equations (13.27) through (13.30), with parameter vectors $\boldsymbol{\theta}_Y = \{\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1\}$ and $\boldsymbol{\theta}_Q = \{\boldsymbol{\beta}_2, \boldsymbol{\beta}_3, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3\}$ (ignoring error distribution parameters), assuming we could observe $\{\bar{S}^s_n, \bar{D}^d_n, \bar{Y}^s_n, \bar{Y}^d_n\}$. Let $\hat{\boldsymbol{\theta}}_Y$ and $\hat{\boldsymbol{\theta}}_Q$ be the estimates that maximize $L_Y(Y, \pi_Q, \boldsymbol{\theta}_Y)$ and $L_Q(Q, \pi_Y, \boldsymbol{\theta}_Q)$. With these definitions, we can conclude:

THEOREM 13.1: If the reduced forms (13.21) through (13.23) and (13.24) through (13.26) and structural equations (13.27) through (13.30) each satisfy assumptions 13.1 through 13.4, then the MLE's $\{\hat{\pi}_Y, \hat{\pi}_Q, \hat{\boldsymbol{\theta}}_Y, \hat{\boldsymbol{\theta}}_Q\}$ will be strongly consistent for $\{\pi_Y, \pi_Q, \boldsymbol{\theta}_Y, \boldsymbol{\theta}_Q\}$, and the second-stage estimators $\tilde{\boldsymbol{\theta}}_Y$ and $\tilde{\boldsymbol{\theta}}_Q$ that maximize the functions

$$L_Y(Y, \hat{\pi}_Q, \boldsymbol{\theta}_Y),$$
$$L_Q(Q, \hat{\pi}_Y, \boldsymbol{\theta}_Q)$$

will also be strongly consistent for $\boldsymbol{\theta}_Y$ and $\boldsymbol{\theta}_Q$.

PROOF: The proof follows almost directly from Hartley and Mallela (1977). Details of the proof are given in section 13.7.

It would be desirable to derive an expression for the asymptotic distribution of the second-stage estimators $\tilde{\boldsymbol{\theta}}_Y$ and $\tilde{\boldsymbol{\theta}}_Q$. Unfortunately an examination of a Taylor expansion of the normal equations indicates that, unless relatively restrictive conditions are met, an expression for the asymptotic distribution of $\tilde{\boldsymbol{\theta}}_Y$ and $\tilde{\boldsymbol{\theta}}_Q$ will involve terms from the reduced forms as well as the second stage. We can, however, derive easily estimable expressions for the asymptotic distribution of the maximum likelihood estimators $\hat{\boldsymbol{\theta}}_Y$ and $\hat{\boldsymbol{\theta}}_Q$. Using $\hat{\boldsymbol{\theta}}_Y$ as an example, these can be estimated by the inverse of the information matrix of (13.27) through (13.28) as follows:

THEOREM 13.2: If conditions 13.1 through 13.4 are met, then

1. $\sqrt{n}\,(\tilde{\boldsymbol{\theta}}_Y - \boldsymbol{\theta}_Y) \rightarrow N\left(\mathbf{0}, \left[-\frac{\partial^2 G_Y(\boldsymbol{\pi}_Q, \boldsymbol{\theta}_Y)}{\partial \boldsymbol{\theta}_Y \partial \boldsymbol{\theta}_Y'}\right]^{-1}\right)$,

where

$$G_Y(\boldsymbol{\pi}_Q, \boldsymbol{\theta}_Y) = \lim_{n\to\infty} \frac{1}{n} \ln L_Y(Y, \boldsymbol{\pi}_Q, \boldsymbol{\theta}_Y).$$

Furthermore, if conditions for theorem 13.1 hold, then

2. $G_Y(\hat{\boldsymbol{\pi}}_Q, \tilde{\boldsymbol{\theta}}_Y)$ converges to $G_Y(\boldsymbol{\pi}_Q, \boldsymbol{\theta}_Y)$ a.e. the same holding for $\hat{\boldsymbol{\theta}}_Q$.

PROOF: Part 1 follows directly from Hartley-Mallela (1977). Part 2 follows from Hartley-Mallela lemma 10 (showing uniform convergence for $G_Y(\cdot)$) and lemma 13.1 of Section 13.7 (Amemiya 1973, lemma 4).

Although the asymptotic distribution of the maximum likelihood estimators is unlikely to be the same as that of $\{\tilde{\boldsymbol{\theta}}_Y, \tilde{\boldsymbol{\theta}}_Q\}$, we could interpret estimates of their standard errors as lower bounds of the standard errors of $\tilde{\boldsymbol{\theta}}_Y$ and $\tilde{\boldsymbol{\theta}}_Q$.

The major advantage of our proposed procedure is computational simplicity and feasibility. Each of the four equation systems—the two reduced forms and the two second-stage models—can be estimated separately, using computer programs for standard switching regression models. An alternative procedure, full information ML, might require estimation of a prohibitively large number of parameters. However, by not using full information ML, we are likely to be losing efficiency as we are ignoring cross-equation restrictions and common system switching points. If cross-equation restrictions are necesssary for full identification of the model, then our procedure, which ignores such restrictions, will be incapable of producing consistent structural parameter estimates.

Our proposed procedure requires the estimation of four switching regression model systems. Since these systems are relatively expensive to estimate, as a final modification we show how one of these runs can be eliminated.

The expectation of the observed dependent variable of a switching regression model, given the exogenous variables, can be computed by a straightforward method. This formula can be used to estimate an instrument for observed debt, Q, from the debt-reduced form. This instrument can then be plugged into (13.11) in place of Q and structural parameters of the Y equation estimated by OLS. This step avoids having to estimate the second-stage Y switching regressions. Formally the instrument is derived from the debt-reduced forms as follows.

Let $\hat{\sigma}_s^{*2}$, $\hat{\sigma}_d^{*2}$, $\hat{\sigma}_{sd}^{*}$ be the variance and covariance estimates of the errors of the reduced forms (13.24) and (13.25), respectively, and let $\hat{\sigma} = \sqrt{\hat{\sigma}_s^{*2} + \hat{\sigma}_d^{*2} - 2\hat{\sigma}_{sd}^{*}}$. Defining $f(\cdot)$ as the standard normal density function, and $F(\cdot)$ as the cumulative normal, a consistent estimate of the expectation of Q_n given $\mathbf{X}_{1n}$, $\mathbf{X}_{2n}$, and $\mathbf{X}_{3n}$ is

$$E(Q_n \mid \mathbf{X}_{1n}, \mathbf{X}_{2n}, \mathbf{X}_{3n}) = F\left(\frac{\hat{\bar{S}}_n^s - \hat{\bar{D}}_n^d}{\hat{\sigma}}\right)\hat{\bar{D}}_n^d + \left(1 - F\left(\frac{\hat{\bar{S}}_n^s - \hat{\bar{D}}_n^d}{\hat{\sigma}}\right)\right)\hat{\bar{S}}_n^s - \hat{\sigma} f\left(\frac{\hat{\bar{S}}_n^s - \hat{\bar{D}}_n^d}{\hat{\sigma}}\right), \qquad (13.32)$$

where $\{\hat{\bar{S}}_n^s, \hat{\bar{D}}_n^d\}$ are estimates of $\{\bar{S}_n^s, \bar{D}_n^d\}$ calculated using the estimated reduced form parameters. It can then be shown that under very general conditions, consistent estimates of the structural parameters $\boldsymbol{\beta}_1$ and $\boldsymbol{\alpha}_1$ can be obtained by substituting $E(Q_n \mid \mathbf{X}_{1n}, \mathbf{X}_{2n}, \mathbf{X}_{3n})$ for Q_n in the original linear structural equation (13.11) and running OLS.

We propose to estimate the parameters of the model we derived in section 13.3, using a procedure based on the methods just described. We discuss our specific application and present empirical estimates in section 13.5.

13.5 The Evidence

The estimation procedure used for our model follows directly from the procedure outlined in the previous section. We can briefly summarize the process. The first step is the estimation of switching regression reduced form equations for each of the three endogenous variables—autos, other durables, and consumer debt. Parameters of the reduced forms are estimated by maximum likelihood assuming a Hartley-Mallela switching regression model with correlated equation errors. A complication of the reduced form estimation is the requirement that the sign of the debt variable in both the auto and durable structural equations be known.

The second step of the procedure is to construct instrumental variables for the three endogenous variables. Although all are not used, potentially three different instruments could be computed for each reduced form: (1) an instrument for the observed dependent variable computed using the formula given in equation (13.32), (2) an instrument for the unobserved demand dependent variable formed from the demand equaton, and (3) an instrument for the unobserved supply dependent variable.

The final step of the procedure is the estimation of structural equation parameters. Both the auto and other durable equations are estimated by OLS, using instruments for the observed dependent variables (instrument form 1). Although OLS yields consistent parameter estimates, we note that the form of equation (13.32) implies that OLS standard errors are unlikely to be consistent.

The equations of particular interest to this chapter are the structural equations for debt. Unfortunately, since debt demand is assumed to be a function of the same exogenous variables as durable and auto demand, the structural equation for debt demand is not fully identified. Thus we are forced to estimate a reduced form for the second-stage debt demand equation. The debt supply equation, however, is identified.

Estimates of the debt supply and demand equations are computed by maximizing a switching regression likelihood function of the exogenous variables and supply instruments for autos and other durables. As shown in section 13.4, these estimates will be strongly consistent.

Maximum likelihood estimates (or quasi-maximum likelihood in case of the second-stage equations) of the switching regression parameters are computed using a Davidon-Fletcher-Powell iterative procedure with two criteria for convergence. Each element of the vector of first derivatives and the change in the log likelihood function value is required to be within a preset tolerance of zero. As a check on the quality of convergence the negative of the matrix of log likelihood second derivatives is computed and, if invertible, used as an estimate of the information matrix and for standard errors. We note, however, that for the second stage, the matrix of second derivatives will not yield estimates of our actual coefficient standard errors. A correct interpretation is that they yield consistent estimates of the standard errors of the maximum likelihood estimators and hence can be considered lower bounds.

Before we report the results of our estimation, it may be useful to show a few sample statistics. Sample means of several key variables broken down by race are given in table 13.2. One statistic that stands out is the substantially higher average debt holdings for whites than for blacks or Hispanics. This difference, which supports Bell's (1974) evidence, is mirrored in auto and durable holdings but is not nearly as apparent in income.

The first step of the estimation was the three switching regression reduced forms, parameter estimates of which are given in section 13.8. It

Table 13.2
Sample statistics (means broken down by race)

	Total	White	Black	Spanish-speaking
Number	604	264	201	139
Age (years head)	39.6	41.0	38.2	39.1
Education (years head)	8.9	10.0	8.6	7.5
Number in family	6.3	5.8	7.1	6.2
Family weekly income ($)	141.0	147.0	141.0	130.0
Value automobile ($)	466.0	606.0	374.0	334.0
Value other durables ($)	991.0	1118.0	916.0	858.0
Consumer debt ($)	855.0	1191.0	627.0	546.0

was assumed that debt would enter both the auto and other durable equations with a positive sign, thus each switching regression dependent variable was assumed to be a minimum. This assumption is consistent with the view that consumers are concerned primarily with their net wealth position. The assumption is supported by positive coefficients later estimated for the second-stage structural equations. To check the robustness of this assumption, we re-estimated each of the reduced forms, assuming the dependent variables were maximums not minimums. However, the relevant second-stage coefficients again turned out to be positive, supporting our original assumption.

Satisfactory convergence of the reduced forms was achieved for the auto and debt equations but not for durable goods. Although the functional values and log likelihood first derivatives met our criteria of convergence, the negative of the matrix of second derivatives would not invert. The problem was caused by second derivatives associated with the equation error correlation coefficient, estimated to be an implausible 0.999. We computed standard errors for the durable goods equation therefore using the Davidon-Fletcher-Powell approximation to the information matrix.

The final step of the estimation procedure were the structural equations. Coefficient estimates of the OLS auto and durable goods equations are given in tables 13.3 and 13.4. Both equations show the expected positive signs for the debt instrument variable indicating that a 1 dollar increase in debt increases the stock of autos by 24 cents and other durables by 21 cents. Coefficients on race accounted for at most a $41 difference in stock levels. This contrasts with the figures of table 13.2, which showed blacks on average held only 62 percent of the auto stock and 82 percent of other

Table 13.3
Structural equation for automobile stock (100's $)

Variable		Coefficient
$\hat{Q}$	Debt instrument	0.235
A_2	Age 26–35	2.231
A_3	Age 36–45	1.875
A_4	Age 46–55	1.080
A_5	Age $\geq$ 56	1.020
E_2	Education 8–11	0.278
E_3	Education $\geq$ 12	0.525
F_1	Total in family	−0.452
F_2	Number of adults	0.250
HL_1	Bad health head	0.444
HL_2	Bad health spouse	1.394
H	Homeownership	3.110
I_1	Total weekly income	0.030
I_4	Earned/total income	2.032
I_5	Variance of earnings	0.023
N_1	Neighborhood median income	−0.002
N_3	Fraction neighborhood black	0.079
N_4	Neighborhood housing/income	−0.005
X	Negative income tax	0.945
B	Black	−0.051
SP	Spanish-speaking	0.346
	Constant	−4.712
		$R^2 = 0.1451$

Table 13.4
Structural equation for other durables (100's $)

Variable		Coefficient
$\hat{Q}$	Debt instrument	0.213
A_2	Age 26–35	−0.292
A_3	Age 36–45	−0.906
A_4	Age 46–55	−0.952
A_5	Age ≥ 56	−0.642
E_2	Education 8–11	0.011
E_3	Education ≥ 12	0.364
F_1	Total in family	0.226
F_2	Number of adults	0.019
HL_1	Bad health head	−0.938
HL_2	Bad health spouse	0.656
H	Homeownership	0.852
I_1	Total weekly income	0.019
I_4	Earned/total income	−0.389
I_5	Variance of earnings	0.024
N_1	Neighborhood median income	0.001
N_3	Fraction neighborhood black	−1.039
N_4	Neighborhood housing/income	−0.002
X	Negative income tax	−0.554
B	Black	−0.414
SP	Spanish-speaking	−0.378
	Constant	4.820
		$R^2 = 0.1539$

durable stock as whites. A portion of this difference must be attributable to demographic differences and importantly to different expected holdings of debt.

The equations of particular interest to this chapter are the structural equations for consumer debt presented in table 13.5. Satisfactory convergence was achieved on all counts. The most striking feature of the demand equation is the significant age effects. As might be predicted by a life-cycle model, debt demand declines significantly with age. Demand increases with bad health, number in family, and homeownership as might be expected but decreases with variance in income and increases with the ratio of earned to total income. Coefficients for both black and Spanish-speaking households are positive and significant. This provides support for the view of Kain and Quigley (1972) that blacks would have higher rather than lower demands for short-term debt.

Coefficients of the supply equation contrast noticeably with those of the demand equation. Age effects are far less pronounced. Total in family enters negatively rather than positively. Garnishing and residence of less than one year enter negatively as might be expected. Somewhat surprisingly neighborhood income above the poverty level enters negatively and ever-on-welfare enters positively. The instruments for autos and durables both show expected positive signs. The larger coefficients for autos indicate firms are willing to finance a higher fraction of auto purchases than other durables. Finally, coefficients for both black and Spanish-speaking households are both large and negative, indicating these groups will be constrained at significantly lower debt levels than comparable whites. The racial coefficients are offset by the large positive constant term. This has the net effect of assigning all white observations to the demand regime and all but one of the black and Spanish-speaking observations to the supply constrained regime. Thus our procedure has in effect split the sample along racial lines.

Table 13.5
Second-stage equations for consumer debt (100's $)

Variable		Coefficient	Approximate standard errors
Demand equation			
A_2	Age 26–35	−8.768	4.668*
A_3	Age 36–45	−13.951	4.729***
A_4	Age 46–55	−16.333	4.736***
A_5	Age ≥ 56	−20.538	5.586***
E_2	Education 8–11	2.939	2.200
E_3	Education ≥ 12	3.180	2.452
F_1	Total in family	0.621	0.451
F_2	Number of adults	−0.606	0.992
HL_1	Bad health head	4.986	2.187**
HL_2	Bad health spouse	3.564	2.340
H	Homeownership	1.503	1.622
I_1	Total weekly income	0.068	0.023***
I_4	Earned/total income	3.357	3.648
I_5	Variance of earnings	−0.084	0.039**
N_1	Neighborhood median income	0.009	0.037
N_3	Fraction neighborhood black	−2.932	3.329
N_4	Neighborhood housing/income	−0.002	0.020
X	Negative income tax	−0.456	1.634
B	Black	12.040	4.026***
SP	Spanish-speaking	11.145	5.443**
	Constant	6.932	8.497

Table 13.5
(continued)

Variable		Coefficient	Approximate standard errors
Supply equation			
$\hat{A}$	Auto instrument	0.483	0.256*
$\hat{D}$	Durable instrument	0.277	0.730
A_1	Age < 26	2.190	2.244
A_5	Age ≥ 56	−3.371	2.095
F_1	Total in family	−0.307	0.312
H	Homeownership	0.840	1.876
I_2	Head week earnings	0.015	0.022
I_3	Spouse week earnings	−0.038	0.030
I_6	Dummy income > $100	1.792	1.492
I_7	Dummy ever welfare	0.616	1.213
N_2	Dummy neighborhood income > poverty	−0.978	0.993
B	Black	−55.064	32.580*
SP	Spanish-speaking	−56.870	32.580*
C_1	Average job tenure	−0.006	0.102
C_2	Dummy residence < 1 year	−0.308	1.415
C_3	Dummy garnished/repossessed	−0.869	4.034
	Constant	57.018	32.622*
	Standard devation demand error	11.436	0.513***
	Standard devation supply error	7.578	0.318***
	Correlation of errors	0.251	0.260
	Log of the likelihood function = − 2193.0		

Note: * Significant at 10 percent level; ** significant at 5 percent level; *** significant at 1 percent level.

13.6 Qualifications and Evaluations

We have asked a lot of our procedure. We asked it to estimate parameters of two debt equations based only on the slim information that observed debt will be the minimum of two unobserved supply and demand variables and the assumption that equation errors are normally distributed. Our problem is compounded by the fact that we use micro data subject to large stochastic noise.

A priori the above arguments suggest it is not likely we would achieve what we sought—identification of the parameters of low-income debt supply and debt demand functions. The actual empirical evidence leaves us no more optimistic. Our procedure may have done little more than divide the sample along racial lines. The implied conclusion—that debts of virtually all whites are demand determined and of all blacks and Hispanics supply constrained—seems implausible at best. We suspect that our trouble in obtaining convergence for the other durable reduced form equation was caused in part by the severity of the sample separation. Since virtually all sample observations had estimated regime probabilities of 0 or 1, there was little information to estimate the correlation of equation error terms.

Before discarding our results as a purely methodological exercise, however, let us recall the empirical inconsistency generated by Shinkel (1976) and Bell (1974). Bell found evidence, supported by our sample, that low-income blacks have substantially less debt than comparable whites. If Bell's evidence is true, and if Shinkel's conclusion that race plays a small role in direct credit screening is correct, then either (1) blacks must demand less debt or (2) blacks are indirectly screened. On this question we believe our results have some merit. It is noteworthy that the particular sample division estimated by our procedure separates white into the demand regime and blacks into the supply regime. As we suggest in section 13.5, parameter estimates suggest that debt holdings within the white population are allocated in a manner consistent with our view of cross-sectional demand. Similarly debt holdings within the black population appear to be allocated consistently with our view of a supply function. Thus although we have doubts about specific parameter estimates of our model, we believe that our evidence supports the view that observed differences in black-white debt holdings are a supply not a demand phenomena.

13.7 Appendix: Proof of Theorem 13.1

The proof of theorem 13.1 follows almost directly from lemmas proved by Hartley and Mallela (1977). We require two additional lemmas, the first of which is taken directly from lemma 4 of Amemiya (1973). Each of these lemmas is stated in terms of single parameters, although they are easily generalized to parameter vectors.

LEMMA 13.1: Let $f_n(\omega, \theta)$ be a measurable function on a measurable space $\boldsymbol{\Omega}$ and for ω in $\boldsymbol{\Omega}$ a continuous function for θ in a compact set $\boldsymbol{\Theta}$. If $f_n(\omega, \theta)$ converges to $f(\theta)$ a.e. uniformly for all θ in $\boldsymbol{\Theta}$, and if $\hat{\theta}_n(\omega)$ converges to θ^0 a.e., then $f_n[\omega, \hat{\theta}_n(\omega)]$ converges to $f(\theta^0)$ a.e. Proof of this lemma 13.1 is given in Amemiya (1973).

LEMMA 13.2: Let $f_n(\omega, \theta_1, \theta_2)$ be a measurable function on a measurable space $\boldsymbol{\Omega}$ and for each ω in $\boldsymbol{\Omega}$ a continuous function for $\{\theta_1, \theta_2\}$ in a compact set $\boldsymbol{\Theta}$. If

1. $f_n(\omega, \theta_1, \theta_2)$ converges to $f(\theta_1, \theta_2)$ uniformly a.e. $\forall \omega \in \Lambda$ such that $\mathscr{P}(\Lambda) = 1$,
2. $f(\theta_1^0, \theta_2^0)$ is a unique maximum for $\{\theta_1, \theta_2\}$ in $\boldsymbol{\Theta}$,
3. $\hat{\theta}_{n1}(\omega)$ converges to θ_1^0 a.e. $\forall \omega \in \Lambda$ such that $\mathscr{P}(\Lambda) = 1$.

Then $\hat{\theta}_{n2}(\omega)$, such that

$$f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)) = \sup_{\theta_2 \in \boldsymbol{\theta}} f_n(\omega, \hat{\theta}_{n1}(\omega), \theta_2)$$

converges to θ_2^0 a.e.

PROOF: The proof parallels that of Hansen (1979, theorem 1). By definition

$$f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)) \geq f_n(\omega, \hat{\theta}_{n1}(\omega), \theta_2^0).$$

Taking limits, it follows from lemma 13.1 and conditions 1 and 3, that

$$\lim_{n \to \infty} \inf f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)) \geq f(\theta_1^0, \theta_2^0).$$

$\forall \omega \in \bar{\Lambda}$ such that $\mathscr{P}(\bar{\Lambda}) = 1$.

We now assume that lemma 13.2 is not true and show that this leads to a contradiction. If conditions 1, 2, and 3 hold, but $\hat{\theta}_2(\omega)$ does not converge to θ_2^0, a.e., then there exists $\bar{\Lambda}$, a subset of $\bar{\Lambda}$, $\mathscr{P}(\bar{\Lambda}) > 0$ such that for some

$\delta > 0$ there exists an unbounded subset of the positive integers, $\bar{\mathbf{I}}(\omega)$, such that for all $\omega \in \bar{\mathbf{\Lambda}}$

$$|\hat{\theta}_{n2}(\omega) - \theta_2^0| > \delta, \quad \text{for } n \in \bar{\mathbf{I}}(\omega),\ \omega \in \bar{\mathbf{\Lambda}}.$$

It also follows that

$$|\{\hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)\} - \{\theta_1^0, \theta_2^0\}| > \delta, \quad \text{for } n \in \bar{\mathbf{I}}(\omega),\ \omega \in \bar{\mathbf{\Lambda}}.$$

Consider a region $\mathbf{S}$ about $\{\theta_1^0, \theta_2^0\}$, such that $[\theta_1, \theta_2] \in \mathbf{S}$ implies $|\{\theta_1^0, \theta_2^0\} - \{\theta_1, \theta_2\}| > \delta$, then condition 2 implies

$$\inf\{f(\theta_1^0, \theta_2^0) - f(\theta_1, \theta_2)\} > 0, \quad \text{for } [\theta_1, \theta_2] \in \mathbf{S}.$$

Thus there exists $\varepsilon > 0$ such that

$$\{f(\theta_1^0, \theta_2^0) - f(\hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega))\} > 2\varepsilon, \quad \forall n \in \bar{\mathbf{I}}(\omega),\ \omega \in \bar{\mathbf{\Lambda}}.$$

However, condition 1 implies that, given 2ε, there exists an $n'(\omega)$ such that

$$|f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)) - f(\hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega))| < \varepsilon, \quad \forall n > n'(\omega),\ \omega \in \bar{\mathbf{\Lambda}}.$$

Thus

$$\{f(\theta_1^0, \theta_2^0) - f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega))\} > \varepsilon, \quad \forall n > n'(\omega) \text{ and } \in \bar{\mathbf{I}}(\omega),\ \omega \in \bar{\mathbf{\Lambda}};$$

hence

$$f(\theta_1^0, \theta_2^0) \geq \varepsilon + \lim_{n \to \infty} \inf f_n(\omega, \hat{\theta}_{n1}(\omega), \hat{\theta}_{n2}(\omega)), \text{ for all } \omega \in \bar{\mathbf{\Lambda}},$$

which is a direct contradition. ■

The proof of theorem 13.1 consists of demonstrating that the second-stage estimators $\tilde{\theta}_Y$ and $\tilde{\theta}_Q$ satisfy the conditions of lemma 13.2. We demonstrate this as follows. Using $\tilde{\theta}_Q$ as an example, define

$$\theta_1^0 = \{\pi_Y\},$$
$$\theta_2^0 = \{\theta_Q\},$$
$$\hat{\theta}_{n1} = \{\hat{\pi}_Y\},$$
$$\hat{\theta}_{n2} = \{\tilde{\theta}_Q\}.$$

Hartley-Mallela (1977, theorem 4) prove that if conditions (13.1) through (13.4) of section 13.4 are satisfied for (13.21) through (13.26), $\{\hat{\pi}_Y, \hat{\pi}_Q\}$ will be strongly consistent for $\{\pi_Y, \pi_Q\}$. This is sufficient to establish condition 3 of lemma 13.2. Furthermore define

$$f_n^1(\omega, \theta_1, \theta_2) = \ln\left[\frac{\prod_{t=1}^{n} g_t^1(\omega, \theta_1, \theta_2)}{\prod_{t=1}^{n} g_t^1(\omega, \theta_1^0, \theta_2^0)}\right]^{1/n},$$

$$f_n^2(\omega, \theta_1) = \ln\left[\frac{\prod_{t=1}^{n} g_t^2(\omega, \theta_1)}{\prod_{t=1}^{n} g_t^2(\omega, \theta_1^0)}\right]^{1/n},$$

where $g_t^1(\cdot)$ and $g_t^2(\cdot)$ are the likelihood functions of the tth observation of (13.29) and (13.30) and (13.21) through (13.23), respectively. Given conditions 1 through 4 of section 13.4 are satisfied, Hartley-Mallela prove lemmas that imply

1. $f_n(\omega, \theta_1, \theta_2) = f_n^1(\omega, \theta_1, \theta_2) + f_n^2(\omega, \theta_1)$ is measurable and continuous for a compact parameter space (Hartley-Mallela, lemmas 1 and 3),
2. $f_n(\cdot)$ converges uniformly a.e. (Hartley-Mallela, lemmas 8 and 9 and Tchebyschoff's theorem),
3. $\lim_{n\to\infty} f_n^2(\cdot)$ has a unique maximum at θ_1^0, and $\lim_{n\to\infty} f_n^1(\cdot)$ has a unique maximum at θ_2^0 given $\theta_1 = \theta_1^0$ (follows immediately from Hartley-Mallela, theorem 1). This also implies that $\lim_{n\to\infty} f_n^1(\cdot)$ achieves a maximum at θ_1^0, θ_2^0 (since 13.29 and 13.30 are just rearrangements of 13.24 through 13.26). Thus $\lim_{n\to\infty} f_n(\cdot)$ has a unique maximum at θ_1^0, θ_2^0.

Noting that maximizing $f_n(\cdot)$ is equivalent to maximizing the sample likelihood function of (13.29) and (13.30), parts 1, 2, and 3, are sufficient to establish conditions 1 and 2 of lemma 13.2. Thus theorem 13.1 is proved.

13.8 Appendix: Empirical Reduced Form Equations

Tables 13.6 through 13.8 report the fitted reduced form equations for the study of consumer credit in section 13.5.

Table 13.6
Reduced form for automobile stock (100's $)

Variable		Coefficient	Standard error
Demand-constrained regime			
A_2	Age 26–35	−0.033	2.900
A_3	Age 36–45	−2.676	2.923
A_4	Age 46–55	−3.755	2.976
A_5	Age ≥ 56	−6.908	3.545*
E_2	Education 8–11	−0.095	1.416
E_3	Education ≥ 12	−0.557	1.572
F_1	Total in family	−0.608	0.289**
F_2	Number of adults	0.929	0.680
HL_1	Bad health head	1.404	1.457
HL_2	Bad health spouse	2.695	1.530*
H	Homeownership	4.286	1.127***
I_1	Total weekly income	0.064	0.015***
I_4	Earned/total income	2.336	2.477
I_5	Variance of earnings	−0.027	0.027
N_1	Neighborhood median income	0.011	0.024
N_3	Fraction neighborhood black	−0.720	2.242
N_4	Neighborhood housing/income	0.003	0.013
X	Negative income tax	0.101	1.097
B	Black	28.757	6.712***
SP	Spanish-speaking	4.830	2.072**
	Constant	−5.106	5.343

Table 13.6
(continued)

Variable		Coefficient	Standard error
Supply-constrained regime			
A_2	Age 26–35	1.611	2.484
A_3	Age 36–45	2.849	2.610
A_4	Age 46–55	1.018	2.895
A_5	Age ≥ 56	4.291	3.420
E_2	Education 8–11	1.963	1.193*
E_3	Education ≥ 12	3.203	1.673*
F_1	Total in family	−0.120	0.260
F_2	Number of adults	−0.663	0.653
HL_1	Bad health head	1.712	1.678
HL_2	Bad health spouse	2.513	1.613
H	Homeownership	1.852	1.474
I_2	Head week earnings	0.038	0.025
I_3	Spouse week earnings	0.057	0.028**
I_4	Earned/total income	−0.315	4.243
I_5	Variance of earnings	0.050	0.028*
I_6	Dummy income > $100	−0.517	1.686
I_7	Dummy ever welfare	−0.289	1.192
N_1	Neighborhood median income	0.009	0.032
N_2	Dummy neighborhood income > poverty	−0.681	1.607
N_3	Fraction neighborhood black	0.966	2.066
N_4	Neighborhood housing/incone	−0.019	0.014
X	Negative income tax	1.649	1.380
B	Black	−129.237	240.941
SP	Spanish-speaking	−124.205	240.941
C_1	Average job tenure	0.070	0.111
C_2	Dummy residence < 1 year	−1.501	1.046
C_3	Dummy garnished/repossessed	1.371	1.967
	Constant	125.058	240.941
	Standard deviation demand error	7.948	0.328***
	Standard deviation supply error	7.802	0.429***
	Correlation of errors	−0.124	0.121
	Log of the likelihood function = − 2102.6		

Note: * Significant at 10 percent level; ** significant at 5 percent level; *** significant at 1 percent level.

Table 13.7
Reduced form for other durables (100's $)

Variable		Coefficient	Standard error
Demand-constrained regime			
A_2	Age 26–35	−3.215	2.249
A_3	Age 36–45	−5.181	2.402**
A_4	Age 46–55	−6.311	2.345***
A_5	Age ≥ 56	−4.759	3.224
E_2	Education 8–11	0.684	1.401
E_3	Education ≥ 12	1.648	1.475
F_1	Total in family	0.157	0.438
F_2	Number of adults	0.055	0.147
HL_1	Bad health head	−0.383	1.123
HL_2	Bad health spouse	2.149	2.139
H	Homeownership	0.260	0.866
I_1	Total weekly income	0.033	0.019*
I_4	Earned/total income	−0.040	0.925
I_5	Variance of earnings	0.017	0.023
N_1	Neighborhood median income	−0.008	0.005*
N_3	Fraction neighborhood black	−2.656	2.122
N_4	Neighborhood housing/income	−0.000	0.001
X	Negative income tax	−1.779	0.931*
B	Black	5.727	2.222***
SP	Spanish-speaking	5.167	2.554**
	Constant	10.405	3.484***

Table 13.7
(continued)

Variable		Coefficient	Standard error
Supply-constrained regime			
A_2	Age 26–35	−0.067	0.069
A_3	Age 36–45	−0.550	0.627
A_4	Age 46–55	−0.025	0.617
A_5	Age ≥ 56	−0.370	0.712
E_2	Education 8–11	0.583	0.391
E_3	Education ≥ 12	0.045	0.057
F_1	Total in family	0.255	0.115**
F_2	Number of adults	0.043	0.038
HL_1	Bad health head	0.322	0.858
HL_2	Bad health spouse	0.046	0.075
H	Homeownership	1.804	0.968*
I_2	Head week earnings	0.035	0.011***
I_3	Spouse week earnings	0.031	0.013**
I_4	Earned/total income	−1.267	1.991
I_5	Variance of earnings	0.020	0.017
I_6	Dummy income > $100	0.086	0.085
I_7	Dummy ever welfare	0.716	0.364**
N_1	Neighborhood median income	0.009	0.008
N_2	Dummy neighborhood income > poverty	−0.028	0.022
N_3	Fraction neighborhood black	−0.026	0.028
N_4	Neighborhood housing/income	−0.011	0.007
X	Negative income tax	0.637	0.402
B	Black	−55.162	39.458
SP	Spanish-speaking	−54.442	39.465
C_1	Average job tenure	0.037	0.048
C_2	Dummy residence < 1 year	0.980	0.692
C_3	Dummy garnished/repossessed	4.949	1.252***
	Constant	56.951	39.560
	Standard deviation demand error	6.666	0.299***
	Standard deviation supply error	3.396	0.139***
	Correlation of errors	0.999	0.050***
	Log of the likelihood function = − 1790.8		

Note: * Significant at 10 percent level; ** significant at 5 percent level; *** significant at 1 percent level.

Table 13.8
Reduced form for consumer debt (100's $)

Variable		Coefficient	Standard error
Demand-constrained regime			
A_2	Age 26–35	−8.830	4.695*
A_3	Age 36–45	−13.975	4.762***
A_4	Age 46–55	−16.368	4.766***
A_5	Age ≥ 56	−20.562	5.650***
E_2	Education 8–11	2.840	2.349
E_3	Education ≥ 12	3.170	2.563
F_1	Total in family	0.612	0.464
F_2	Number of adults	−0.624	1.019
HL_1	Bad health head	5.017	2.220**
HL_2	Bad health spouse	3.486	2.391
H	Homeownership	1.434	1.648
I_1	Total weekly income	0.069	0.024***
I_4	Earned/total income	3.199	3.693
I_5	Variance of earnings	−0.086	0.040**
N_1	Neighborhood median income	0.010	0.037
N_3	Fraction neighborhood black	−3.207	3.472
N_4	Neighborhood housing/income	−0.001	0.020
X	Negative income tax	−0.411	1.669
B	Black	12.849	4.900***
SP	Spanish-speaking	11.191	5.848*
	Constant	7.155	8.615

Table 13.8
(continued)

Variable		Coefficient	Standard error
Supply-constrained regime			
A_2	Age 26–35	−0.696	2.268
A_3	Age 36–45	−1.320	2.400
A_4	Age 46–55	−1.838	2.741
A_5	Age ≥ 56	−3.559	3.139
E_2	Education 8–11	1.577	1.087
E_3	Education ≥ 12	0.950	1.459
F_1	Total in family	−0.151	0.256
F_2	Number of adults	−0.710	0.630
HL_1	Bad health head	1.339	1.549
HL_2	Bad health spouse	0.559	1.480
H	Homeownership	2.526	1.440*
I_2	Head week earnings	0.023	0.024
I_3	Spouse week earnings	−0.011	0.027
I_4	Earned/total income	3.095	3.954
I_5	Variance of earnings	0.049	0.028*
I_6	Dummy income > $100	2.662	1.515*
I_7	Dummy ever welfare	0.384	1.117
N_1	Neighborhood median income	−0.020	0.029
N_2	Dummy neighborhood income > poverty	−0.127	1.468
N_3	Fraction neighborhood black	2.422	1.963
N_4	Neighborhood housing/income	−0.010	0.012
X	Negative income tax	0.203	1.269
B	Black	−60.550	71.557
SP	Spanish-speaking	−59.406	71.524
C_1	Average job tenure	0.084	0.106
C_2	Dummy residence < 1 year	−0.840	0.960
C_3	Dummy garnish/repossess	1.271	1.971
	Constant	63.105	71.677
	Standard deviation demand error	11.446	0.491***
	Standard deviation supply error	7.471	0.332***
	Correlation of errors	0.273	0.149*
	Log of the likelihood function = − 2187.1		

Note: * Significant at 15 percent level; ** significant at 5 percent level; *** significant at 1 percent level.

References

Amemiya, T. 1973. Regression Analysis When the Dependent Variable is Truncated Normal. *Econometrica*. 41; 997–1016.

Amemiya, T. 1974. A Note on the Fair and Jaffee Model. *Econometrica*. 42; 759–762.

Anderson, R. 1976. Durable Goods, Savings, and Consumer Credit. Research paper 148. Graduate School of Business, Columbia University.

Avery, B. 1977. Information Restrictions and Equal Credit Opportunity Legislation. Working paper. Carnegie-Mellon University.

Cain, G. C. 1967. Unemployment and Labor Force Participation of Secondary Workers. *Industrial Labor Relations Review*. 20; 275–297.

Dolde, W., and J. Tobin. 1971. Wealth, Liquidity, and Consumption. In *Consumer Spending and Monetary Policy; The Linkages*. Boston: Federal Reserve Bank of Boston, pp. 99–160.

Fair, R. C., and D. M. Jaffee. 1972. Methods of Estimation for Markets in Disequilibrium. *Econometrica*. 40; 497–514.

Friedman, M. 1957. *A Theory of the Consumption Function*. Princeton: Princeton University Press, National Bureau of Economic Research.

Hansen, L. P. 1979. Consistency Theorems for a General Class of Econometric Estimators. Working paper. Carnegie-Mellon University.

Harris, D. G. 1974. Credit Rationing at Commercial Banks. *Journal of Money Credit and Banking*. 5; 227–240.

Hartley, M. J. 1973. On the Estimation of Markets in Disequilibrium. Discussion paper 275. Economics Department, State University of New York at Buffalo.

Hartley, M. J., and P. Mallela. 1977. The Asymptotic Properties of a Maximum Likelihood Estimator for a Model of Markets in Disequilibrium. *Econometrica*. 45; 1205–1220.

Jaffee, D. M., and F. Modigliani. 1969. A Theory and Test of Credit Rationing. *American Economic Review*. 69; 850–872.

Kain, J. F., and J. M. Quigley. 1972. Housing Market Discrimination, Homeownership, and Savings Behavior. *American Economic Review*. 62; 263–277.

Laffont, T. T., and R. Garcia. 1977. Disequilibrium Econometrics for Business Loans. *Econometrica*. 45; 1187–1204.

Maddala, G. S., and F. D. Nelson. 1974. Maximum Likelihood Methods for Markets in Disequilibrium. *Econometrica*. 42; 1013–1030.

Metcalf, C. E. 1977. Consumption Behavior: Implications for a Permanent Program. In *The New Jersey Income-Maintenance Experiment, Volume III: Expenditures, Health, and Social Behavior; and the Quality of the Evidence*, ed. Harold W. Watts and Albert Rees. New York: Academic Press, pp. 93–111.

Mincer, J. 1960. Labor Supply, Family Income, and Consumption. *American Economic Review Proceedings*. 50; 574–583.

Peterson, R. L. 1976. The Impact of General Credit Restraint on the Supply of Commercial Bank Consumer Installment Credit. *Journal of Money Credit and Banking*. 7; 527–535.

Schmidt, P. 1979. Constraints on the Parameters in Simultaneous Probit and Tobit Models. In *Structural Analysis of Discrete Data: With Econometric Applications* ed. Charles F. Manski and Daniel McFadden. Cambridge, Mass.: MIT Press.

Sealey, C. W. 1978. Credit Rationing in the Commercial Loan Market: Estimates of a Structural Model under Conditions of Disequilibrium. Department of Finance, Arizona State University.

Shinkel, B. A. 1976. *The Effects of Limiting Information in the Granting of Credit.* Ph.D. dissertation. Purdue University.

Tobin J. 1957. Consumer Debt and Spending: Some Evidence from Analysis of a Survey. Comsumer Installment Credit, part 2, vol. 1, Board of Governors of the Federal Reserve System. Washington: Government Printing Office, pp. 521–545.

Tobin, J. 1972. Wealth, Liquidility, and the Propensity to Consume. In *Human Behavior in Economic Affairs: Essays in Honor of George Katona*, ed. B. Strumpel, et al. Amsterdam: Elsevier, pp. 36–56.

Watkins, T. H. 1975. The Time Allocation of Consumption under Debt Limitations. *Southern Economic Journal.* 42; 61–68.

Watts, H. W., and J. Tobin 1960. Consumer Expenditures and the Capital Account. *Proceedings of the Conference on Consumption and Savings*, vol. 2, ed. Irwin Friend and Robert Jones. Philadelphia: University of Pennsylvania Press, pp. 1–48.

Index