University of California, Berkeley
Economics 201B
Spring 2006 Final Exam–May 19, 2006

Instructions: You have three hours to do this exam. The exam is out of a total of 300 points; allocate your time accordingly. **Please write your solutions to Parts I and II in separate bluebooks; you get five points for doing this.**

Part I

1. (80 points) Define or state and *briefly* discuss the importance of each of the following within or for economic theory:
   (a) Index Theorem
   (b) First Welfare Theorem in the Arrow-Debreu Economy
   (c) Implicit Function Theorem
   (d) Arrow security
2. (75 points) From the proof of the Debreu-Gale-Kuhn-Nikaido Lemma:
   (a) Define the correspondence that is used in the proof.
   (b) Show that any fixed point of the correspondence must lie in the interior of the price simplex.
   (c) Show that any interior fixed point must be an equilibrium price.

Part II

3. (90 points) Consider an exchange economy with $I = 2$ agents and $L = 2$ goods, with fixed endowment profile $\omega \gg 0$. Agent 2 has a continuous "demand" function $D_2(p) \in \mathbf{R}^2_+$ which satisfies Walras' Law, but agent 2 need not be "rational," i.e. we don't assume that $D_2(p)$ maximizes a preference relation. Agent 1 is "rational", with a parameterized Cobb-Douglas utility function

$$u_\alpha(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \text{ where } \alpha \in (0,1)$$

Let $f(p, \alpha)$ be the excess demand for this economy at the price $p \in \Delta^0$.

(a) Show that for every $\alpha \in (0,1)$ and every $\varepsilon > 0$, there is a two-person exchange economy with "rational" agents such that the excess demand of the economy equals $f(p, \alpha)$ on $\{p \in \Delta : p_1 \geq \varepsilon, p_2 \geq \varepsilon\}$.

(b) Show that for every $\alpha \in (0,1)$, there exists p^*_α such that $f(p^*_\alpha, \alpha) = 0$.

(c) Supose in addition that D_2 is C^1. Show that except for a set of α of Lebesgue measure zero, the economy is regular.

4. (50 points) Consider an exchange economy with $H = 2$ consumers and L goods, with social endowment $\bar{\omega} \in \mathbf{R}^L_{++}$. Suppose the two consumers a, b have continuous, strongly monotone, and strictly convex preferences. In this question, we will also consider the n-fold replica of this economy. In the n-fold replica, there are $2n$ agents, of whom n (referred to as type a agents) have preferences and endowments identical to those of agent a in the original economy, and n (referred to as type b agents) have preferences and endowments identical to those of agent b in the original economy. Consider a Pareto optimal allocation $x = (x_a, x_b)$ of the 2-agent economy.

   (a) Let $x^{(n)}$ denote the n-fold replica of x, in which the type a agents all consume x_a and the type b agents all consume x_b. Show that $x^{(n)}$ is Pareto optimal in the n-fold replica economy.

   (b) Suppose that there is a coalition S that can block $x^{(n)}$ by some x'. Show that the coalition S can also block $x^{(n)}$ by an x'' with the property that all type a members of S are assigned the same consumption by x'', and all type b members of S are assigned the same consumption by x''.