Economics 201b
Spring 2010
Problem Set 7
Due Thursday May 6th 5PM John Zhu's Mailbox 970 Evans

1. Consider a standard asset market as in MWG 19.E: where an asset $r = (r_1, r_2, \ldots, r_S) \in \mathbb{R}^S$ is a return vector of amounts of good 1 across the S states. Assume every return vector is nonnegative and nonzero. Let R be a return matrix of K return vectors (i.e. R is a nonnegative, nonzero $S \times K$ matrix).

   (a) Let $\mu \in \mathbb{R}^S$ be a vector of state multipliers with $\mu \gg 0$. Show that a system of asset prices $q \in \mathbb{R}^K$ where $q^T = \mu \cdot R$ is arbitrage free. In addition, point out why this claim may be false if we only assume the weaker condition $\mu \geq 0$.

   (b) Show that the set Q of *arbitrage free* prices is convex.

   (c) Consider the following set of return vectors

$$r_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \quad r_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad r_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

   Suppose $q_1 = 4$, $q_2 = 5$. What is the set of values q_3 may take so that the corresponding system of asset prices is arbitrage free?

2. Consider an economy with 2 individuals, 2 goods, and 2 states ($I = L = S = 2$). Suppose that the social endowment $\bar{\omega}_s$ of goods in state s is

$$\bar{\omega}_1 = (1, 2) \qquad \bar{\omega}_2 = (3, 3)$$

   and the two agents' utilities are

$$U_i(x_{11i}, x_{21i}, x_{12i}, x_{22i}) = x_{11i}x_{21i} + x_{12i}x_{22i}$$

   where the first coordinate denotes the good, and the middle coordinate denotes the state. Now consider the following two securities (see Lecture 14 for an example)

$$S_1 \text{ pays } \begin{cases} (1,0) & \text{in state 1} \\ (0,1) & \text{in state 2} \end{cases} \qquad S_2 \text{ pays } \begin{cases} (3,6) & \text{in state 1} \\ (h,0) & \text{in state 2} \end{cases}$$

   (a) What is the unique Radner equilibrium spot price vector $p^* = (p_1^*, p_2^*) \in \Delta^o \times \Delta^o$?

   (b) For what values of h does p^* correspond to a Hart point? From now on fix $h = 0$, what is the corresponding payoff vector R_i for each security S_i?

   (c) Let (q_1^*, q_2^*) be a price vector for the securities in a Radner equilibrium. What is $\frac{q_2^*}{q_1^*}$?

   (d) Suppose $\omega_{11} = (0, 1)$, $\omega_{21} = (2, 0)$ (and therefore $\omega_{12} = (1, 1)$, $\omega_{22} = (1, 3)$). What are the agents' Radner equilibrium portfolios z_1^*, $z_2^* \in \mathbb{R}^2$ and consumption plans x_1^*, $x_2^* \in \mathbb{R}^4$?