Economics 201b
Spring 2010
Problem Set 2
Due Thursday April 1

1. **Competitive Equilibrium(-a) When Preferences Are Kinked.** Recently there have been a surge in decision theory models that are non-differentiable in nature. For example, popular models incorporating loss aversion in prospect theory, or ambiguity aversion as illustrated by Ellsberg Paradox, have kinked indifference curves.

   In this exercise we are going to take a reduced form of these preferences and examine the implications of "kinkiness" on equilibrium prices and allocations in our simplest 2 × 2 exchange economy, where agents' utility functions are $\forall i \in \{1, 2\}$,

$$U_i(x_{i1}, x_{i2}) = \begin{array}{ll} \sqrt{x_{i1}} + \frac{1}{2}\sqrt{x_{i2}} & \text{if } x_{i1} \leq x_{i2} \\ \frac{1}{2}\sqrt{x_{i1}} + \sqrt{x_{i2}} & \text{if } x_{i1} > x_{i2}. \end{array} \tag{1}$$

   (a) Suppose the initial endowments are $\omega_i = (4, 4)$, $\forall i \in \{1, 2\}$. Draw the Edgeworth box for this economy. Find the Pareto optimal allocations. Verify that the initial endowment is an equilibrium allocation. Find the supporting equilibrium price(s).

   (b) Now suppose the endowments are instead $\omega'_1 = (5, 3)$, $\omega'_2 = (3, 5)$. Find the individual and market excess demand functions (notice that the utility function is not differentiable). Find the competitive equilibrium prices and allocations. Is there a unique equilibrium?

   (c) Now suppose the endowments are instead $\omega''_1 = (8, 2)$, $\omega''_2 = (3, 5)$. Repeat the calculations that you have done in (b). (Note that the Edgeworth box is different in this case). Comment on how the kinkiness of preferences affect the size of competitive equilibria.

   (d) Does the First Welfare Theorem holds in this economy? What about the Second Welfare Theorem?

2. **More Fun with Offer Curves!** Consider simple two-person, two-good economy in which agents' utility functions are given by

$$U_1(x_{11}, x_{21}) = \min\{x_{11}, x_{21}\}, \text{ and } U_2(x_{12}, x_{22}) = \min\{4x_{12}, x_{22}\}. \tag{2}$$

   and endowments are $\omega'_1 = (30, 0)$, $\omega'_2 = (0, 20)$.

   (a) If neither agents can have negative consumption of either good, what is Walrasian equilibrium?

(b) Now suppose the first agent starts only with 10 units of good 1 instead of 30 and none of the second. What is Walrasian equilibrium in this case? Explain briefly your results. *Hint: be sure to find all Walrasian equilibria.*

(c) Suppose that an agent decides to throw away part of her endowment to change the equilibrium prices in the economy. Can agent be better off in the new equilibrium than in the equilibrium with the original endowment? Provide an example and explain. (The example does not have to be analytic, however, it must be described *clearly* and *coherently*).

3. **Equilibrium with "Bads."** Consider an exchange economy that contains two consumers with utility function of the form $\forall i \in \{1, 2\}$:

$$U_i(x_{1i}, x_{2i}) = x_{1i}(4 - x_{2i}) \tag{3}$$

defined over consumption set $[0, 5] \times [0, 3] \subset \mathbb{R}^2_+$. Notice that the second commodity is "bad." Endowments are given by $\omega_1 = (1, 3)$ and $\omega_2 = (3, 1)$.

(a) Show that feasible allocation $x$ is Pareto optimal if and only if $x_{11}+x_{21} = 4$.

(b) Compute excess demand functions and find the Walrasian equilibrium. Illustrate it with Edgeworth box diagram.

(c) What happens to the Walrasian equilibrium if the first consumer has the right to dump all of her endowment of the second commodity onto the second consumer?

(d) What happens to the Walrasian equilibrium if there is an *ad valorem* tax $t$ on any sale of good 2 paid by the seller, which is then shared equally between two agents?

4. **Importance of Assumptions.** Give examples of the following, and illustrate them using an Edgeworth box. Please be *clear* and *precise.*

(a) A Pareto optimal allocation that can't be sustained as a Walrasian equilibrium with transfers.

(b) A Walrasian equilibrium that is not Pareto optimal (Please do not use externalities).

5. **Computing the Transfers.** Consider again the exchange economy from the question 1 of the problem set 1: a two-person, two-good exchange economy where the agents' utility functions are $U_1(x_{11}, x_{21}) = x_{11}x_{21}$ and $U_2(x_{12}, x_{22}) = x_{12}x_{22}$, and the initial endowments are $\omega_1 = (1, 3)$ and $\omega_2 = (3, 1)$. Show directly that every interior Pareto optimal allocation in this economy is a price equilibrium with transfers by finding the associated prices and transfers.