Economics 201B–Second Half

Lecture 12-4/22/10

Justifying (or Undermining) the Price-Taking Assumption

- Many formulations: Core, Ostroy's No Surplus Condition, Bargaining Set, Shapley-Shubik Market Games (noncooperative), other noncooperative games

- Core is the most commonly used. The *core* is the set of all allocations such that no coalition (set of agents) can *improve on* or *block* the allocation (make all of its members better off) by seceding from the economy and only trading among its members.

- Core is institution-free; no mention of prices.

- "Core convergence" means roughly that

  For economies with a large number of agents, core allocations are "approximately Walrasian."

- "Approximately Walrasian" means different things in different contexts, depending on what we are willing to assume.

- *Three motivations for the study of the core*:

  - *Walrasian allocations lie in the core*: Important strengthening of First Welfare Theorem, under same minimal assumptions as First Welfare Theorem.

    * (Positive): Strong stability property of Walrasian equilibrium: no group of individuals would choose to upset the equilibrium by recontracting among themselves.

    * (Normative): If distribution of initial endowments is equitable, no group is treated unfairly at a core allocation. Since Walrasian allocations lie in the core, this is a Group Fairness Property of Walrasian Equilibrium.

- *Core Convergence strengthens Second Welfare Theorem*
    * Second Welfare Theorem says every Pareto Optimum is a Walrasian Equilibria with Transfers.
    * Core convergence asserts that core allocations of large economies are *nearly* Walrasian *without* transfers.
    * One version states that core allocations can be realized as *exact* Walrasian equilibrium with *small* income transfers.
    * *Strong "unbiasedness" property of Walrasian equilibrium*
        · Restricting to Walrasian outcomes does not narrow possible outcomes beyond narrowing occurring in the core.
        · (Normative) No hidden implications for welfare of different groups beyond equity issues in the initial endowment distribution.
        · (Normative) Assuming distribution of endowments is equitable, any allocation that is far from Walrasian will not be in the core, and hence will treat some group unfairly.

- *Core Convergence justifies Price-Taking, Core Nonconvergence suggests Price-Taking is Implausible*:
    * The definition of Walrasian equilibrium contains (hidden in plain sight) assumption that economic agents act as price-takers.

* In real markets, we see prices used to equate supply and demand, but this does not guarantee Walrasian outcome.
* Agents possessing market power may choose to supply quantities different from the competitive supply for the prevailing price, thereby altering that price and leading to a non-Pareto Optimal outcome.
* If outcome is not Walrasian, Welfare Theorems, Existence, Determinacy would have limited implications for real economies.
* (Positive) Core convergence and nonconvergence allows us to identify situations in which price-taking is more or less reasonable.
* Edgeworth defined core in 1881, in *Mathematical Psychics*, an ambitious book developing microeconomic theory in mathematical terms.
* Edgeworth criticized Walras, thought the core, not the set of Walrasian equilibria, was best positive description of outcomes from market mechanism.
  - In particular, the definition of the core does not impose the assumption of price-taking behavior made by Walras.
  - Furthermore, if any allocation not in the core arose, some group would find it in its interests to recontract. Edgeworth thus argues that the core is the significant positive equilibrium concept.
  - If core is correct positive concept, core convergence justifies price-taking. Core convergence says all trade takes place at almost a single price. Agent who tries to bargain

cannot influence prices much, and cannot change outcome much (argument more compelling with stronger convergence notions).

  · If core is correct positive concept, core nonconvergence undermines price-taking. Edgeworth himself argued that in real life, the presence of large firms leads to failure of price-taking.

- **Definition 1** In an exchange economy, a *coalition* is a set

$$S \subseteq \{1, \ldots, I\}$$

A coalition S *blocks* or *improves on* an exact allocation x by x' if

$$\sum_{i \in S} x'_i = \sum_{i \in S} \omega_i$$

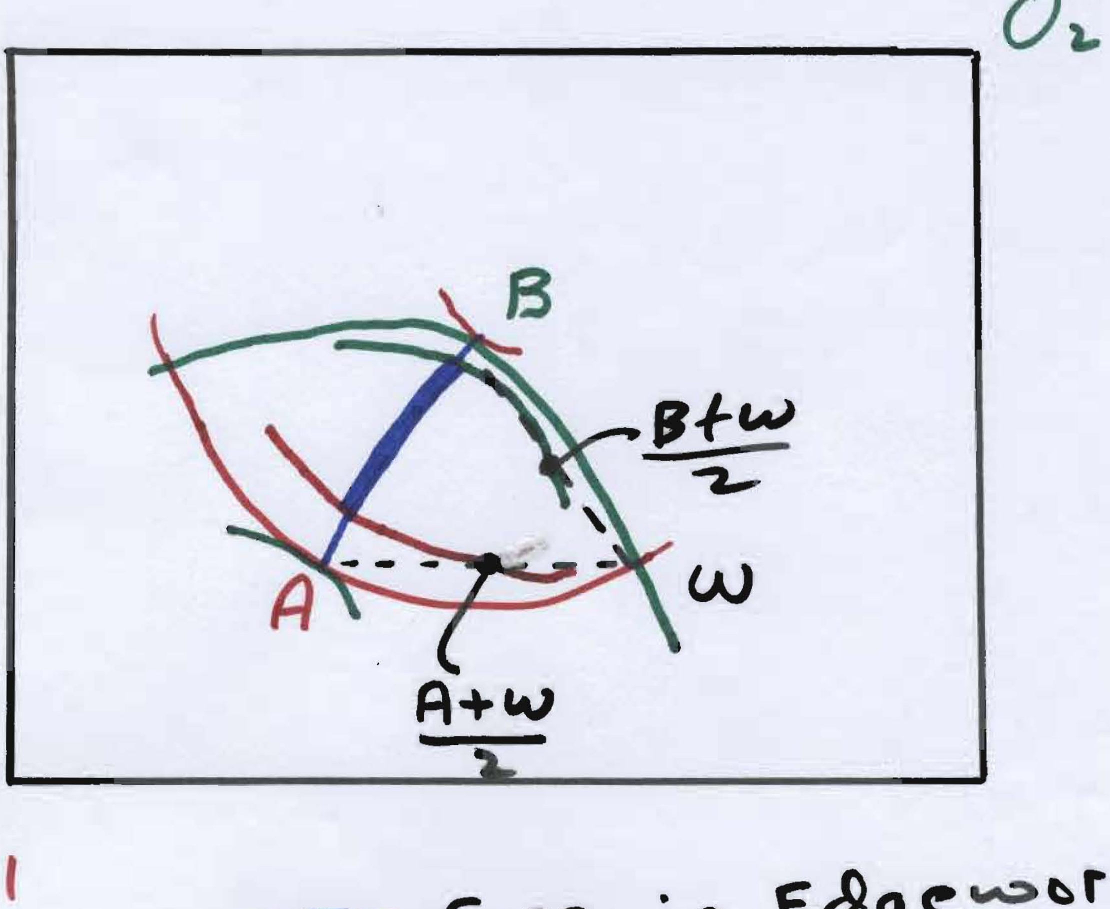
and

$$\forall_{i \in S} \; x'_i \succ_i x_i$$

The *core* is the set of all exact allocations which cannot be improved on by any nonempty coalition.

- Notice we follow MWG and require $x'_i \succ_i x_i$ for all $i \in S$; this is analogous to the definition of weakly Pareto Optimal. *Natural*: status quo should be focal, need strict improvement to join a coalition to upset the status quo.

- Notice that the definition of blocking by a coalition does not specify what happens to the individuals outside the coalition. One might imagine individuals not in the blocking coalition making a counterproposal to some of those in the blocking coalition; the Bargaining Set takes these counterproposals into account.



Core in Edgeworth Box

Core in 2-fold replica (Two agents of each type)

- It is a common mistake to ask, at a core allocation, what coalition(s) are active. A core allocation is defined by the fact that *no* coalition can defeat it.

- **Theorem 2** *In an exchange economy, every core allocation is weakly Pareto Optimal.*

  Proof: If x is not weakly Pareto Optimal, then there exists x',

$$\sum_{i=1}^{I} x'_i = \bar{\omega}, \ x'_i \succ_i x_i$$

  Then $S = \{1, \ldots, I\}$ improves on x by x', so x is not in the core.■

- **Theorem 3 (Strong First Welfare Theorem)** *In an exchange economy, every Walrasian Equilibrium lies in the core.*

  Proof: Suppose (p^*, x^*) is a Walrasian Equilibrium. If x^* is not in the core, there exists $S \subseteq I$, $S \neq \emptyset$ and $x'_i (i \in S)$ such that

$$\sum_{i \in S} x'_i = \sum_{i \in S} \omega_i, \ \forall i \in S \ x'_i \succ_i x^*_i$$

  Since $x^*_i \in D_i(p^*)$,

$$p^* \cdot x'_i > p^* \cdot \omega_i$$

  so

$$\begin{aligned} p^* \cdot \sum_{i \in S} x'_i &= \sum_{i \in S} p^* \cdot x'_i \\ &> \sum_{i \in S} p^* \cdot \omega_i \\ &= p^* \cdot \sum_{i \in S} \omega_i \end{aligned}$$

  but

$$\sum_{i \in S} x'_i = \sum_{i \in S} \omega_i$$

  contradiction. Therefore, x^* is in the core.■

Theorem 4 *Suppose we are given an exchange economy with L commodities, I agents, and preferences $\succ_1, \ldots, \succ_I$ satisfying weak monotonicity (if $x \gg y$, then $x \succ_i y$) and the following* free disposal *condition:*

$$x \gg y,\ y \succ_i z \quad \Rightarrow x \succ_i z.$$

If x is in the core, then there exists $p \in \Delta$ such that

$$\frac{1}{I}\sum_{i=1}^{I} |p \cdot (x_i - \omega_i)| \ \leq\ \frac{2L}{I} \max\{\|\omega_1\|_\infty, \ldots \|\omega_I\|_\infty\} \qquad (1)$$

$$\frac{1}{I}\sum_{i=1}^{I} |\inf\{p \cdot (y - x_i) : y \succ_i x_i\}| \ \leq\ \frac{4L}{I} \max\{\|\omega_1\|_\infty, \ldots \|\omega_I\|_\infty\} \qquad (2)$$

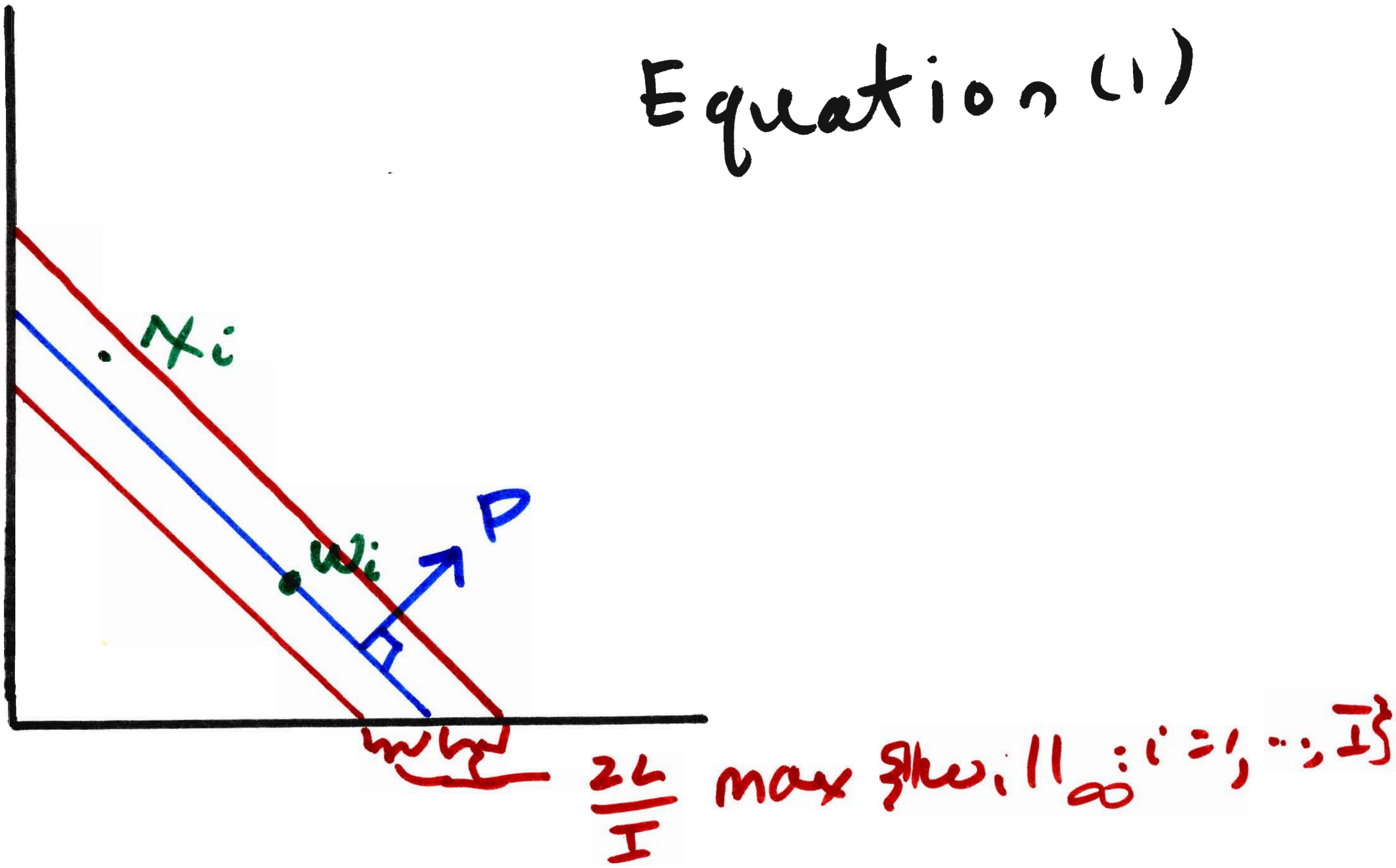
where $\|x\|_\infty = \max\{|x_1|, \ldots, |x_L|\}$.

- Equation (1) says that trade occurs almost at the price p, and that each x_i is almost in the budget set.
- Equation (2) says that the price p almost supports $\succ_i$ at x_i.
- If we knew the left sides of Equations (1) and (2) were *zero*, then

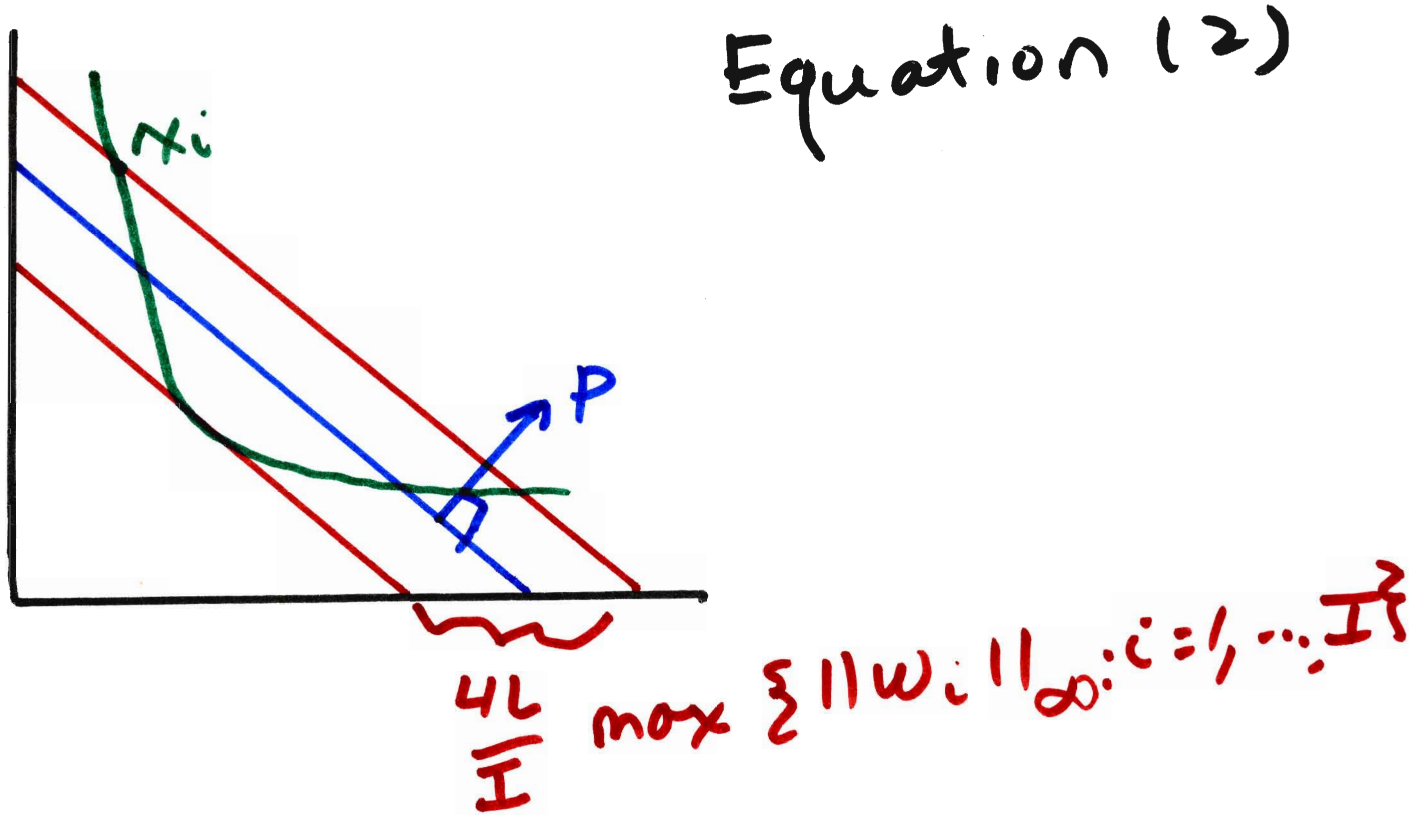
$$p \cdot (x_i - \omega_i) = 0 \quad \Rightarrow \quad x_i \in B_i(p)$$
$$y \succ_i x_i \quad \Rightarrow \quad p \cdot y \geq p \cdot \omega_i$$

so x is a Walrasian quasiequilibrium! Thus, every core allocation satisfies a perturbation of the definition of Walrasian Equilibrium: agent i's consumption need not lie in his/her budget set, but it can't be far outside; anything strictly preferred neeed not be outside the budget set, but it can't be far below the budget frontier.

Equation (1)



Equation (2)



Outline of Proof: Follow the proof of the Second Welfare Theorem.

- Suppose x is in the core. Define

$$\begin{aligned} B_i &= \{y - \omega_i : y \succ_i x_i\} \cup \{0\} \\ &= (\{y : y \succ_i x_i\} \cup \{\omega_i\}) - \omega_i \\ B &= \sum_{i=1}^{I} B_i \end{aligned}$$

  The first term in the definition of B_i corresponds to members of a potential improving coalition; for accounting purposes, we assign members outside the coalition their endowments. Note that B_i is *not* convex, even if $\succ_i$ is a convex preference.

- *Claim*: If x is in the core, then

$$B \cap \mathbf{R}^L_{--} = \emptyset$$

  Suppose $z \in B \cap \mathbf{R}^L_{--}$. Then

$$\exists_{z_i \in B_i} \ z = \sum_{i=1}^{I} z_i$$

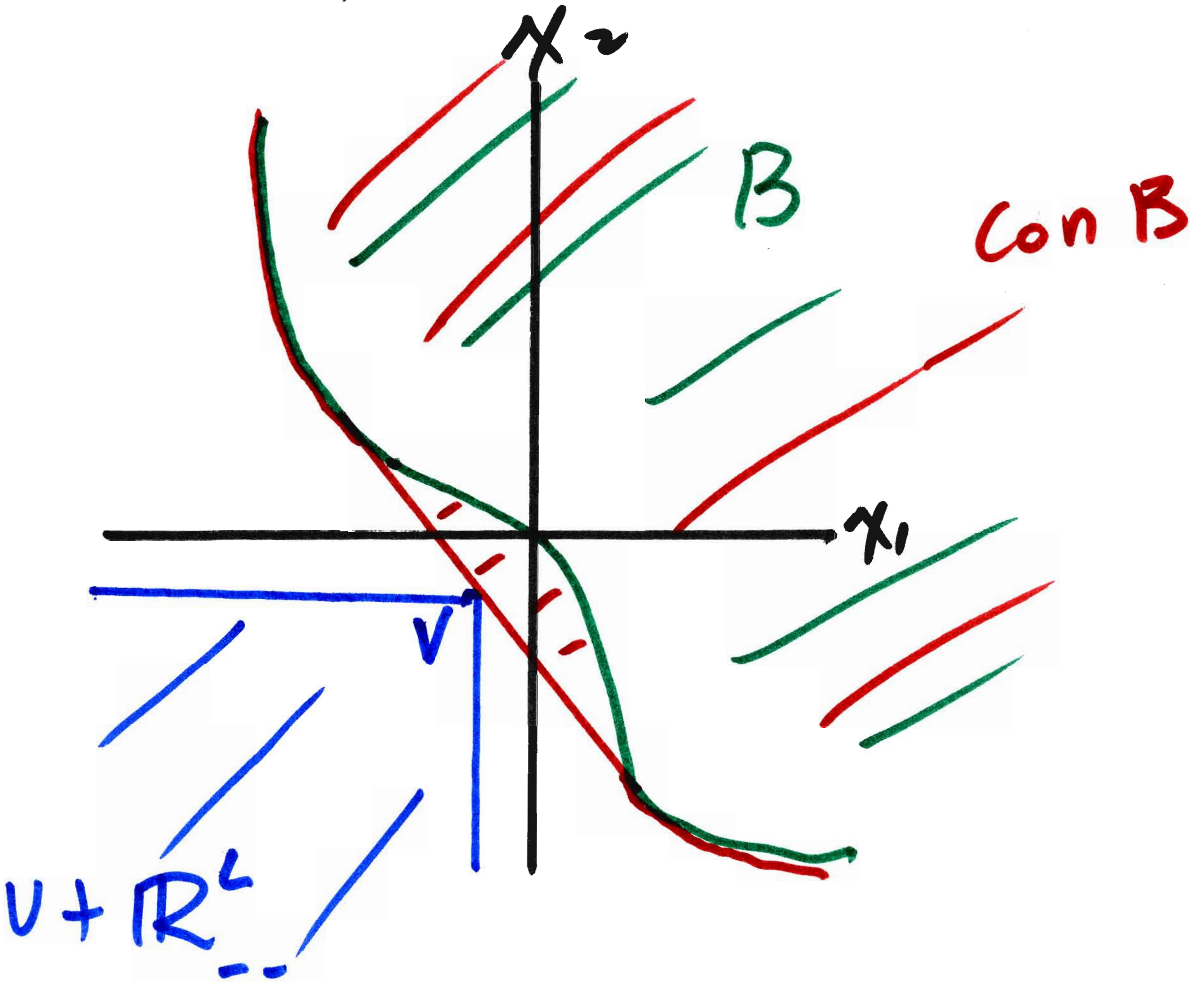
  Let

$$S = \{i : z_i \neq 0\}$$

  Since $z \ll 0$, $S \neq \emptyset$. For $i \in S$, let

$$\begin{aligned} x'_i &= \omega_i + z_i - \frac{z}{|S|} \\ x'_i &\gg \omega_i + z_i \succ_i x_i \text{ (definition of } B_i) \\ x'_i &\succ_i x_i \text{ (free disposal)} \\ \sum_{i \in S} x'_i &= \sum_{i \in S} \omega_i + \sum_{i \in S} z_i - z \\ &= \sum_{i \in S} \omega_i + z - z \\ &= \sum_{i \in S} \omega_i \end{aligned}$$

  so S can improve on x by x', so x is not in the core.



x_2
x_1
B
Con B
v
$v + \mathbb{R}^L_{--}$

- Let

$$v = -L(\max_{i=1,\ldots,I} ||\omega_i||_\infty, \ldots, \max_{i=1,\ldots,I} ||\omega_i||_\infty)$$

Claim:

$$(\text{con } B) \cap \left(v + \mathbf{R}^L_{--}\right) = \emptyset$$

If $z \in \text{con } B$, by the Shapley-Folkman Theorem, and relabelling the agents, we may write

$$\begin{aligned} z &= \sum_{i=1}^{I} z_i \\ z_i &\in \text{con } B_i \, (i = 1, \ldots, I), \\ z_i &\in B_i \ (i \notin \{1, \ldots, L\}) \end{aligned}$$

Choose

$$\hat{z}_i = \begin{cases} 0 & \text{if } i = 1, \ldots, L \\ z_i & \text{if } i = L+1, \ldots, I \end{cases}$$

Then $\sum_{i=1}^{I} \hat{z}_i \in B$ so

$$\sum_{i=1}^{I} \hat{z}_i \not\ll 0$$

If $z \ll v$, then

$$\begin{aligned} \sum_{i=1}^{I} \hat{z}_i &= \sum_{i=1}^{L} 0 + \sum_{i=L+1}^{I} z_i \\ &\leq \sum_{i=1}^{L} (\omega_i + z_i) + \sum_{i=L+1}^{I} z_i \\ & \quad (\text{since } z_i \in \text{con } B_i, \, \omega_i + z_i \in \text{con } (\omega_i + B_i) \\ & \quad \subset \text{con } \mathbf{R}^L_+ = \mathbf{R}^L_+) \\ &= \sum_{i=1}^{L} \omega_i + \sum_{i=1}^{I} z_i \\ &= \sum_{i=1}^{L} \omega_i + z \end{aligned}$$

$$
\begin{aligned}
&\ll \sum_{i=1}^{L} \omega_i + v \\
&\leq 0
\end{aligned}
$$

so

$$B \cap \mathbf{R}^L_{--} \neq \emptyset$$

a contradiction which proves the claim.

- By Minkowski's Theorem, there exists $p \neq 0$ such that

$$\sup p \cdot \left(v + \mathbf{R}^L_{--}\right) \leq \inf p \cdot (\text{con } B)$$

If $p_\ell < 0$ for some ℓ, then

$$
\begin{aligned}
\sup p \cdot \left(z + \mathbf{R}^L_{--}\right) &= +\infty \\
\inf p \cdot (\text{con } B) &\leq 0
\end{aligned}
$$

contradiction, so $p > 0$ and we can normalize $p \in \Delta$.

$$
\begin{aligned}
\inf p \cdot B &\geq \inf p \cdot (\text{con } B) \\
&\geq p \cdot v \\
&= -L \max \{\|\omega_1\|_\infty, \ldots, \|\omega_I\|_\infty\}
\end{aligned}
$$

- Adapt the remainder of the proof of the Second Welfare Theorem (requires a few tricks).