University of California, Berkeley
Economics 201A
Fall 2000 Final Examination

Instructions: You have three hours to do this examination. The exam is out of a total of 300 points; allocate your time accordingly. Please write your solution to each question in a **separate** bluebook.

1. (75 points) Define or state and *briefly* discuss the importance of each of the following within or for economic theory:

   (a) Brouwer's Fixed Point Theorem

   (b) a theorem on the existence of approximate Walrasian equilibrium, when preferences are nonconvex

   (c) Index Theorem

   (d) a theorem concerning the cores of exchange economies with many consumers

   (e) incomplete markets

2. (75 points) Tom Hanks is the sole owner of a firm with access to the production technology

$$Y = \{(y_1, y_2) : y_1 \leq 0,\ y_2 \leq \sqrt{-y_1}\}$$

   Hanks' endowment is $(1, 0)$. Hanks has the Cobb-Douglas utility function $u(x_1, x_2) = \sqrt{x_1 x_2}$. There are no other firms or consumers in the economy. Find a Walrasian equilibrium (p^*, x^*, y^*).

3. (150 points) Consider a pure exchange economy with aggregate endowment $\overline{\omega} \gg 0$ in which every consumer has a "rational," continuous, strictly convex and strongly montone preference. A social planner imposes a price-dependent income transfer scheme of the following form:

$$T_i(p) = \alpha \left(\frac{p \cdot \overline{\omega}}{I} - p \cdot \omega_i \right) \tag{1}$$

where $\alpha \in [0, 1]$ is a constant.[1] The budget set for agent i is $B_i(p) = \{x \in \mathbf{R}^L_+ : p \cdot x \leq p \cdot \omega_i + T_i(p)\}$; the demand is $D_i(p) = \{x \in B_i(p) : x' \in B_i(p) \Rightarrow x \succeq_i x'\}$. Let $z(p) = \sum_{i=1}^{I} D_i(p) - \overline{\omega}$.

(a) Show that for every $\alpha \in [0, 1]$, there exists p^* such that $z(p^*) = 0$.

(b) Show that if $z(p^*) = 0$ and x^* is the allocation given by $x_i^* = D_i(p^*)$, then x^* is Pareto optimal. You may do this in either of two ways:

   i. Assume the First Welfare Theorem as stated in class. You will need to explain why it applies to this situation, with a price-dependent income transfer.

   ii. Adapt the proof of the First Welfare Theorem to this situation. If you choose this approach, you will get full credit for proving that x^* is Pareto optimal in the weak sense, that there is no exact allocation x' such that $x'_i \succ_i x_i^*$ for all $i = 1, \ldots, I$.

(c) Suppose the preferences are such that demand is a C^1 function of price and α. Using the transversality theorem, show that for almost all (α, ω), the resulting economy is regular.

(d) Suppose there are exactly two consumers in the economy. What does the Second Welfare Theorem tell you about the ability to achieve a given Pareto optimum using a transfer of the type described in Equation (1)? *Hint*: you will need to consider $\alpha \in \mathbf{R}$, not just $\alpha \in [0, 1]$.

[1] This is a bit like an income tax, in the sense that the tax charged is proportional to the value of each person's endowment; the revenue generated by the tax is then rebated in equal amounts to each individual.

(e) Suppose the social planner imposes the following modified transfer scheme

$$T_i'(p, x) = \alpha \left(p \cdot (\omega_i - x_i)_+ - \frac{\sum_{j=1}^{I} p \cdot (\omega_j - x_j)_+}{I} \right)$$

where x is an allocation and $(y_+)_\ell = \max\{y_\ell, 0\}$.[2] Let $B_i'(p, x)$, $D_i'(p, x)$ and $z'(p, x)$ be defined as above, substituting T_i' for T_i. If $z'(p^*, x^*) = 0$ and $x_i^* = D_i'(p^*, x^*)$, does it follow that x^* is Pareto optimal?

[2]This is closer to the income tax in practice in the United States, in that the tax applies to the income generated by the *sale* of your endowment, but the consumption of your own endowment (for example, living in a house you own that *could* be rented out, but isn't; or the devotion of time to leisure activities) is not taxed. As before, the revenue generated by the tax is rebated in equal amounts to each individual.

***Note to me: the last question (except for the part about the modified transfer T_i') is just like a situation in which each person is given endowment $\alpha\frac{\overline{\omega}}{I}+(1-\alpha)\omega_i$. To see this, note that this endowment is always exactly on the budget frontier for every price. Hence, existence and first welfare theorem follow immediately from the same results in an exchange economy. Generic regularity follows from the fact that the measure induced on endowments by $\alpha\frac{\overline{\omega}}{I} + (1-\alpha)\omega_i$ is mutually absolutely continuous with respect to Lebesgue measure.