

**Near-Rational Wage and Price Setting and the Optimal Rates  
of Inflation and Unemployment**

George A. Akerlof, William T. Dickens, and George L. Perry\*

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## **Near-Rational Wage and Price Setting and the Optimal Rates of Inflation and Unemployment**

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Over thirty years ago, in his Presidential Address to the American Economic Association, Milton Friedman [1968] asserted that in the long run the Phillips Curve was vertical at a natural rate of unemployment that could be identified by the behavior of inflation.<sup>1</sup> Unemployment below the natural rate would generate accelerating inflation—above it, accelerating deflation. Five years later the New Classical economists posed a further challenge to the stabilization orthodoxy of the day. In their models with rational expectations, not only was monetary policy unable to alter the long term level of unemployment, it could not even contribute to stabilization around the natural rate (see, for example, Lucas [1973], Sargent [1973]).) The New Keynesian Economics has shown that even with rational expectations small amounts of wage and price stickiness permit a stabilizing monetary policy.<sup>2</sup> But the idea of a natural unemployment rate that is invariant to inflation still characterizes macro modeling and informs policy making.

The familiar empirical counterpart to the theoretical natural rate is the nonaccelerating inflation rate of unemployment, or NAIRU. Phillips curves embodying a NAIRU are estimated using lagged inflation as a proxy for inflationary expectations. NAIRU models appear in most textbooks and estimates of the NAIRU—which is assumed to be relatively constant—are widely used by economic forecasters, policy analysts and policy makers. However the inadequacy of such models has been demonstrated forcefully in recent years as low and stable rates of inflation

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<sup>1</sup>Also see Phelps [1968] for analysis very similar to that of Friedman.

<sup>2</sup>See Akerlof and Yellen (1985) and Mankiw (1985).

have coexisted with a wide range of unemployment rates. If there is a single relatively constant natural rate we should have seen inflation slowing significantly when unemployment was above that rate and rising when it was below. Instead, the inflation rate has remained fairly steady with annual CPI-U inflation ranging from 1.6 percent to 3.0 percent since 1992 while the annual unemployment rate has ranged from 6.8 to 3.9 percent. In this paper we present a model that can accommodate relatively constant inflation over a wide range of unemployment rates.

Another motivation is a recent finding by William Brainard and George Perry (2000). They estimate a Phillips Curve in which all the parameters are allowed to vary over time and find that the coefficient on the proxy for expected inflation in the Phillips Curve has changed considerably while other parameters of that model have been relatively constant. In particular, Brainard and Perry found that the coefficient on expected inflation was initially low in the 50's and 60's, grew in the 70's, and has fallen since then. The model we present below can explain both why the coefficient on expected inflation might be expected to change over time and, to some extent, the time pattern of changes observed by Brainard and Perry.

Our paper also allows an interpretation of the findings of King and Watson (1994) and Fair (2000). Both find a long-run trade-off between inflation and unemployment. In addition, King and Watson find that the amount of inflation that must be tolerated to obtain a given reduction of unemployment rose considerably after 1970. Our model allows a trade-off, but only at low rates of inflation such as those that prevailed in the 50s, 60s and 90s. At higher rates of inflation no trade-off is apparent.

Much of the empirical controversy surrounding the relationship between inflation and unemployment has focused on how people form expectations. This may be neither the most

important theoretical or empirical issue. Instead, this paper suggests that it is not how people form expectations, but how they use them—even whether they use them at all that is the issue. Economists typically assume that economic agents make the best possible use of the information available to them. In contrast, psychologists who study how people make decisions have a different view. They see individuals as acting like intuitive scientists, who base their decisions on simplified abstract models (see Nisbett and Ross [1980]). But these simple intuitive models can be misleading -- sometimes they are incorrect. Psychologists have studied the use of the simplified abstractions, often called mental frames or decision heuristics, and the mistakes that result from them. Economists should not assume absence of cognitive error in economic decisions; nor should they assume that their own models and those of the public exactly coincide.

We propose that there are three important ways in which the treatment of inflation by real world economic agents diverges from the treatment assumed in economic models. First, when inflation is low, a significant number of people may ignore inflation when setting wages and prices. Second, even when they take it into account, they may not treat it as economists would assume. In particular, we hypothesize that the informal use of inflationary expectations in wage and price decisions leads to less than complete projection of anticipated inflation, with consequences for the aggregate relation between inflation and unemployment. Finally, we believe that workers have a different view of inflation from that of trained economists. Workers see inflation as increasing prices and reducing their real earnings and they do not fully, if at all, appreciate that inflation increases the nominal demand for their services. Thus they have a tendency to view the nominal wage increases they receive at low rates of inflation as a sign that their work is appreciated and to be happier in their jobs as a result. They may also be unaware of

the extent to which inflation is increasing the pay available to them in alternative jobs. Even fully rational employers, who must solve the typical efficiency wage problem, can exploit workers' misperceptions by giving nominal wage increases that are less than what would be required if workers fully incorporated inflation into their mental frames.

If *any* of these three departures from the fully rational use of information on inflation are important, then at low rates of inflation prices and wages will be set consistently lower relative to nominal aggregate demand than they would be at zero inflation. As a result, operating the macro economy with a low but positive rate of inflation will permit a higher level of output and employment to be sustained. We will show that at low rates of inflation the behaviors that we posit, which depart from the fully rational decisions of typical economic models, impose very small costs on those who practice them. Since there may be subjective or objective costs associated with fully rational behavior, or because implementing fully rational behavior may require overcoming some perception threshold or behavioral inertia, it is plausible that these small costs may not be enough to induce rational behavior on the part of all economic agents. However, if inflation increases, the costs of being less than perfectly rational about it will also rise, and people will switch their behavior to take inflation into full account. Thus while increasing inflation modestly above zero will permit lower unemployment, there is a rate of inflation above which the sustainable unemployment rate rises as more and more people adopt fully rational behavior. This rate of inflation thus minimizes the sustainable rate of unemployment and yields maximum employment and output. With monopolistically competitive firms and with efficiency wages, workers and firms will be better off at these higher levels of employment and output. The owners of the firms will have higher profits; the workers will have

jobs they were willing to accept. In our model this minimum sustainable rate is also the optimal unemployment rate.

The remainder of the paper proceeds in three steps. First, we describe departures from perfect rationality at low rates of inflation and present some evidence that supports our view. Second, we formally derive our model of near-rational wage and price setting, show that the costs of near rationality are small, derive a short and long-run Phillips Curve from the model, and present a calibration exercise that shows that, even when only a fraction of wages and prices are influenced by near-rational behavior, there can still be substantial long-run gains in employment from moderate, rather than very low or zero, inflation. Finally, we estimate the theoretical model using post-war quarterly US data. The results support the theoretical model and are surprisingly robust.

### **Near-Rational Behavior Towards Inflation**

As noted above, psychologists and economists who study decision-making approach it differently. Psychologists have identified many ways in which real world decision making departs from economic rationality. Here we describe three ways in which we suspect behavior towards inflation departs from the economist's rational model.

First, psychologists suggest that decision makers—far from making the best use of available information—readily ignore potentially relevant considerations and discard potentially relevant information in order to simplify their decision problems. Kahneman and Tversky [1979]

have dubbed this behavior editing.<sup>3</sup> When people “edit” decision problems they rule out less important considerations in order to concentrate on the few factors that matter most. In this regard, real world decision makers are no different from academic economists when they construct models: unimportant factors are ignored in order to concentrate on important factors. In addition to the study of the cognitive process of editing, there is a related literature in the psychology of perception that suggests that items must reach a threshold of salience before they are even perceived (See Gleitman (1996)). Thus, when inflation is low it may be at most a marginal factor in wage and price decisions, and decision makers may ignore it entirely.

We know of no strong evidence either for or against the view that some wage and price setters ignore inflation<sup>4</sup>, but several before us have suggested the occurrence of such behavior. For example, Eckstein and Brinner (1972) based their model of a shifting Phillips Curve on the assumption that inflationary expectations mattered more in determining inflation in the 1970s

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<sup>3</sup>Kunreuther (1978) has used the phenomena of editing to explain why many people do not buy disaster insurance—very low probability events are ignored in decision making. His book presents the results of experiments that demonstrate the phenomena of editing (pp 165-186).

<sup>4</sup>Direct attempts to assess the effects of forecast inflation on wage setting have ignored the indirect effects of inflation through other information that will be correlated with inflation. Such information includes the wages and prices of competitive and complementary goods and factors. Thus the findings that wage and price setters seem to put little weight on inflation (Blinder *et al.* (1998), Levine (1993)) are inconclusive. For this reason we made our own attempt to solicit such information. We sent an e-mail questionnaire to randomly selected members of the American Compensation Association asking them to recommend wage and salary increases in hypothetical situations varying by respondent in a number of different dimensions. The respondents were given the type of information that personnel executives typically use to make recommendations for wage and salary changes. This information included the wage and salary increases of other firms in their labor market over the past year, the desired relative wage and salary position of their firm, expected wage and salary increases of other firms in their labor market for the next year, the increase in the CPI, the difficulty of hiring and retention, their firm’s expected net revenue growth relative to that of their industry and relative to that of the economy as a whole. The mean of expected wage increases by other firms in the sample was increased one-for-one with the rate of inflation. The total effect of changes in inflation on wage and salary increases by individual firms can be seen by regressing the recommended wage and salary increases on the expected wage and salary increases of others and the CPI. The point estimate of the change caused by a one-point change in the CPI in the wages of an individual firm, given that that firm’s changes are representative of other firms facing the same increase in the CPI, is .738. This estimate is obtained by dividing the coefficient on the CPI by one minus the coefficient on the expected wage increases of other firms. Unfortunately, this estimate has a very high standard error so we cannot rule out the possibility that the impact of an increase in expected CPI inflation on wage inflation would be one for one, but the point estimates suggest our view.

than in the 1960s. One major macroeconomics textbook [Blanchard (1999, pp.153-154)] describes the Post War United States Phillips Curve by an early period of low inflation, which was ignored by wage and price setters, and a later period of high inflation, when the coefficient on last period's inflation was close to one. Two of the officials who over the past five years have been most responsible for obtaining the Federal Reserve's goal of price stability have also suggested the possibility of inflation-editing. Former Fed Vice Chairman Alan Blinder, in company with coauthors, Canetti, Lebow and Rudd (1998), has theorized:

A businessman who cannot keep infinite amounts of information in his head may worry about a few important things and ignore the rest. And when nationwide inflation is low, it may be a good candidate for being ignored. Indeed, one prominent definition of 'price stability' is inflation so low that it ceases to be a factor in influencing decisions.

Senate testimony of Federal Reserve Chairman Alan Greenspan seems to suggest a similar view—that at low rates of inflation economic agents may simply ignore it:

By price stability I mean a situation in which households and businesses in making their savings and investment decisions can safely *ignore* the possibility of sustained, generalized price increases or decreases." [See Greenspan (1988, p. 611), italics added].

Second, even when people pay attention to inflation they may not use expectations as economists typically assume. If economic agents used a formal procedure to make wage and price decisions they would first use available information to determine a desired real wage or price change and then add in the amount of inflation they expect between the time they are making the decision and some time during the period over which they expect the price or wage to be in effect. But if they make the decisions intuitively -- subjectively considering a number of factors including inflation simultaneously — there is no reason to expect that the projection will give the appropriate weight to inflation. One decision heuristic, suggested to us by interviews

with compensation professionals, is that information on inflation may simply be averaged along with other factors to arrive at a nominal wage or price increase. This would mean that an increase in inflation would lead to the setting of a higher wage or price, but the effect would be less than one-for-one. Thus less than complete weighting of inflation is the second departure from full rationality that may influence the relationship between inflation and unemployment.

In fact, textbooks for compensation professionals warn against using the formal procedure that economists would imagine was standard. For example, Milkovich and Newman [1984] warn their readers against granting automatic wage and salary increases, including those for the cost of living. Such automatic grants, they say, reduce the funds available to reward employees for performance. Similar thoughts are expressed in the *Handbook* (Rock and Berger [1991], p556 ) of the influential Hay Group of compensation consultants, in which managers are advised to “avoid linking salary movement to changes in the cost of living, because this creates entitlement and reduces the amount of money available to differentiate for performance.”

The third important departure from the hyper-rational model comes from the way workers perceive inflation. Shiller (1997) has documented very large differences between the intuitive models of inflation used by the lay public, most of whom are wage and salary recipients, and the mental accounting of economists who study the effects of inflation scientifically. Wage and salary earners systematically underestimate the effects of inflation on the wages that their employers will want to pay them, even in questionnaires where the effects of inflation are quite explicit, so that it is highly unlikely that inflation is ignored. As a consequence, and especially at moderate rates of inflation when real wages are not perceptibly eroded, workers' job satisfaction may be enhanced by nominal wage increases even if they fail to fully reflect inflation.

There is considerable evidence for this reaction on the part of workers. Economists see inflation as induced by changes in the money supply and thus as having a uniform effect on nominal wages and other prices so that inflation causes no change in real income. In his questionnaire study Shiller has shown that, in contrast, the public has no such expectations. For example, when asked “to imagine how things would be different if the United States had experienced higher inflation over the last five years” (Shiller, 1997, p.21) only 31 percent of his non-economist subjects believed that their nominal income would have been higher than in the absence of inflation. When asked “to evaluate [a variety] of theories about [how] the effects of general inflation on wages and salary relates to your own experience and your own job,” 60 percent of economists, but only 11 percent of the general public elected that “competition among employers will cause my pay to be bid up. I could get outside offers from other employers, and so, to keep me my employer will have to raise my pay too.” A popular answer for the general public (26 percent), in contrast to economists (4 percent), was: “the price increase will create extra profits for my employer who can now sell output for more; there will be no effect on my pay.” (Shiller, 1997, pp.31-32)

The preceding response suggests that the public fails to understand inflation as a general equilibrium phenomena. They believe that inflation will make them poorer because it bids up the prices of the goods they consume, but they fail to appreciate fully, if at all, that inflation will also bid up the prices of other competing factors and other competing workers, thereby resulting in a rise in their own wages and salaries. Thus, according to Shiller (p. 29), the “biggest gripe about inflation” expressed by 77 percent of the general public (but for only 12 percent of economists) was that inflation “hurts my real buying power. It makes me poorer.”

Economists should not be surprised that individuals underestimate the effect of inflation on the demand for their own services. One of the most significant differences between trained economists and the lay public is economists' greater appreciation of general equilibrium. The cognitive difficulty of general equilibrium has been indicated by the fact, noted by the Commission on Graduate Education, that even economics graduate students do not give the correct explanation for why barbers' wages, in the technically-stagnant hair-cutting industry, have risen over the past century [Krueger, 1991, p. 1044]. If economics graduate students fail to appreciate the effects on barbers' opportunity costs from wage increases due to productivity change outside the hair-cutting industry, it would be a stretch to expect the lay public to see that as inflation rises the demand for their services (in nominal dollars) will similarly rise with it.

Findings by Shafir, Diamond and Tversky are consistent with those of Shiller. In one vignette, which they related to respondents, Shafir *et al* draw the contrast between Ann, with a 2 percent nominal salary increase at zero inflation and Barbara, with a 5 percent nominal salary increase at 4 percent inflation. Most respondents correctly identified that Ann would be better off economically, but they also said that Barbara would be happier and less likely to leave her job. This reaction to the vignette suggests that respondents have not *ignored* the inflation, as they would with editing—otherwise Ann would be judged better off economically. But the other answers, favoring Barbara, suggest that they may also underestimate the effect that inflation will have on Barbara's other alternatives thus leading them to conclude that she will be happier and less likely to quit her job.

Unfortunately, the authors have not probed the reasons why respondents believed Barbara should be happier than Ann, but they are responding as if the inflation has not increased her

alternatives by an equal amount. If the wages that she could get on the outside as well as all of the prices that she would be paying had increased by 4 percent then Barbara should be less happy than Ann and also more likely to leave. Our model of inflation, however, suggests a good reason why Barbara should feel happier than Ann and be less likely to quit her job: she does not feel that her alternatives improve at the rate of inflation. Another question by Shiller suggests that the responses obtained to this vignette reflect the true opinion of the American public. He found (p.37) that about half of the US general public — but only 8 percent of economists — think that they would feel more job satisfaction “if their pay went up....even if prices went up as much.”

Neither the vignette by Shafir *et al* nor Shiller’s question deals with the possibility, perhaps on the mind of the public, that the inflation is caused by a supply shock that decreases the real demand for workers rather than a money-neutral demand shock which leaves all demands unchanged in real terms. Of course, if that is really what is on the mind of the public, even when there is a persistent demand induced increase in the rate of inflation, then workers will still have higher job satisfaction with some small amount of inflation than with no inflation.<sup>5</sup> This then is

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<sup>5</sup>The behavior of COLA clauses is consistent with increasing attention being paid to inflation at higher levels, but there are also other explanations for this phenomenon. As inflation rose in the 1970s and 1980s coverage of union workers by COLA’s in the United States increased. In the late 1960s about one quarter of workers involved in collective bargains were covered by COLA clauses; for the inflationary decade from 1975 to 1985 about 60 percent of workers were covered by COLA clauses (Hendricks and Kahn, 1985, 36-37). As inflation fell in the late 1980s the fraction covered fell to 40 percent in 1990 (Holland, 1995, p.176). Such inflation sensitivity of COLAs is consistent with our basic idea that wage and price setters tend to ignore inflation in their wage and price setting when inflation is low, but tend to take it into account in their wage and price setting as inflation rises. But this evidence has at least two other explanations. It is well known (see Ball, Mankiw, and Romer, 1988, p. 56) that the variance of inflation increases with the level. COLAs may increase at higher levels of inflation as insurance against this variance. Furthermore, if at higher rates of inflation a greater fraction of inflation is due to monetary rather than to real shocks, more contracts will be indexed at higher than at lower rates of inflation (see Gray, 1978).

the third way in which we think that near rationality may impact the relation between inflation and unemployment. If higher job satisfaction at low rates of inflation leads to higher morale, less shirking, higher productivity and less turnover, then firms face a different efficiency wage constraint at low rates of inflation than they face at either zero inflation or at high rates of inflation when workers' attitudes towards inflation may become more rational.

### **A Simple Model of Near-Rational Wage and Price Setting**

We now present a simple formal model of the economy that incorporates the behavioral insights we have just described. In the model, some firms' wage and price setters may ignore inflation or firms may be aware of inflation but use it as only one of several factors in setting wages and prices, thus under-weighting it relative to behavior assumed in hyper-rational models. And workers themselves may ignore or under-weight inflation when considering their satisfaction at their current jobs, which in turn affects their productivity. The net effect on unit labor costs of this behavior by workers may or may not be fully factored into firms' wage setting. While the implications of our model for the behavior of the macro economy is not affected by this aspect of firms' behavior, we formally consider the case where firms do not correctly anticipate the effects on worker satisfaction and productivity because this case permits a simple derivation of the profit shortfall a firm experiences from less than fully rational behavior.

The easiest place to begin the model is with its macroeconomic behavior. Income is determined by the quantity theory equation,

$$(1) \quad \bar{p}Y = M,$$

where  $Y$  is real income,  $\bar{p}$  is the average price level in the economy, and  $M$  is the money supply. The usual constant of such quantity theory equations has been normalized to one by choice of units.

The microeconomics of this economy begins with the boiler plate for models with monopolistically competitive firms. There are  $n$  firms in this economy. They divide up the total aggregate demand,  $M/\bar{p}$ , according to the relative prices for their respective goods, so that the demand for the output of an individual firm is of the form:

$$(2) \quad \frac{1}{n} \frac{M}{\bar{p}} \left(\frac{p}{\bar{p}}\right)^{-\beta}$$

where  $p$  is the price charged by a firm for its own product.

This takes us to the first innovation of the model, which occurs in the formulation of productivity and its effect on wages. All of these firms will pay an efficiency wage, which minimizes the unit labor cost of production. Productivity (and also turnover costs) in each firm depends upon the morale of its workers. That morale, in turn, depends upon workers' conception of their outside opportunities, which has two major determinants. The first of these is the rate of unemployment, which determines how easy it would be for an individual worker to obtain another job. The higher the unemployment rate the lower will be the opportunity cost of workers and therefore the higher the morale inside the firm. The second determinant of morale is the workers' perception of the gap between their wage at their own firm and of the wage outside the firm. That perception depends upon the wage being paid by the worker's current firm and her reference wage, which gives her perception of the wages of other workers. Thus the productivity

of the firm will depend also upon both the wage it pays as well as the level of unemployment.

For convenience we shall give productivity the following functional form:

$$(3) \quad P = -A + B \left(\frac{w}{w^R}\right)^\alpha + C u,$$

where  $P$  denotes labor productivity,  $w$  is the wage paid by the firm,  $w^R$  is the reference wage of its workers and  $u$  is the aggregate unemployment rate.  $\alpha$  is chosen in the range  $0 < \alpha < 1$ .

Firms set both prices and wages one period ahead. In so doing they project the effects of inflation on the reference wages of their workers. These reference wages, of course determine the level of wages that a firm should be paying. Totally rational firms will incorporate all of their expected inflation into the reference wage  $w^R$ . In contrast, near-rational firms—and, similarly, fully rational firms whose workers under-weight inflation in  $w^R$ —will incorporate only a fraction of inflation,  $a$ , into their projections of inflation. When  $a$  is zero inflation is totally ignored. In the intermediate range,  $0 < a < 1$ , it is merely underestimated. Thus the reference wage for fully rational workers for the joint wage and price decisions of fully rational firms is

$$(4) \quad w_r^R = \bar{w}_{-1} (1 + \pi^e),$$

where  $\bar{w}_{-1}$  is the average wage paid to all workers in the previous period, and  $\pi^e$  is the expected rate of price inflation. The reference wage for the wage and price setting decision by near-rational firms, which are engaging in cognitive error, will analogously be:

$$(5) \quad w_{nr}^R = \bar{w}_{-1} (1 + a \pi^e),$$

(5) also describes the reference wage for the near-rational employees.

The profit-maximizing choice of the price for both the rational and for the near-rational firm will take the following form. In both cases the prices will be a mark-up over wages,

$$(6) \quad p_j = m \frac{w_j}{P_j}, \quad \text{where } j=r, nr$$

where  $j$  refers both to rational and near-rational firms,. The mark-up factor  $m$  will be  $\beta/(\beta-1)$ .

These maximizing firms will, in turn, establish their wages as a multiple of their respective reference wages, which will differ for rational and for near-rational firms. The efficiency wage paid by each firm-type will minimize its respective unit labor costs,  $w_j/P_j$ . Accordingly, each type of firm will choose, respectively,

$$(7) \quad w_j = \left( \frac{A - Cu}{B(1 - \alpha)} \right)^{1/\alpha} w_j^R, \quad j=r, nr.$$

Near-rational firms set wages that are different from those of fully rational firms, but the difference does not cumulate. The wages of near rational firms are reset relative to their respective reference wage in each and every period. The reference wages for rational and near-rational firms, which are both rising with inflation, differ only by the fraction  $(1+(1-a)\pi^e)/(1+\pi^e)$ . As a result, the difference between wages at the two types of firms will not grow large; indeed, they will be fairly small at low and moderate levels of inflation.

The profits of each type of firm will be revenues net of labor costs. Given the demand function for firms' product (2) and their labor productivity (3), the profits for the two types of firms will be, respectively,

$$(8) \quad \frac{1}{n} \frac{M}{p} \left[ p_j \left( \frac{p_j}{p} \right)^{-\beta} - \left( \frac{p_j}{p} \right)^{-\beta} \frac{w_j}{P_j} \right] \quad j=r, nr.$$

So far the model has described the case where the firm ignores or under-weights inflation, and also the case where the firm is rational, but workers' reference wages are under-indexed. Both situations will give us similar Phillips Curves. In one case near-rational firms will be switching to true rationality as their costs from near rationality mount with high inflation, in the other case the workers will eventually curb their mis-perceptions as inflation rises. But the two hypotheses are slightly different, and at this point we shall take the junction that analyzes the model where the near-rational firms fail to fully take account of inflation in forming  $w^R$ . This route permits an evaluation of the losses by near-rational firms from their failure to correctly perceive the effects of inflation.

Each of the terms  $p_j$ ,  $w_j$ , and  $P_j$  is known relative to the value of the average wage  $\bar{w}_{-j}$ , from (3), (4), (5), (6) and (7) so it is possible to evaluate the relative profits of rational and near-rational firms. Using the profit function (8) along with the assumption that both rational and near-rational firms have correct expectations about inflation, yields a formula for the relative profits of the two types of firm.<sup>6</sup> The relative increase in profits that a near-rational firm could make by becoming a rational firm is given by the loss function (9),

$$(9) \quad L = 1 - \left( \frac{\alpha z}{z^\alpha - 1 + \alpha} \right)^{1-\beta}.$$

where  $z$  is the ratio  $(1 + a\pi)/(1 + \pi)$ . Equation (9) has three implications for this paper, which we

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<sup>6</sup>A slightly more complicated formula will give the relative profits when  $\pi$  is different from  $\pi^e$ .

shall explore in turn.

As the first implication of (9), those who fail to maximize profits either by ignoring inflation ( $a = 0$ ), or taking it into account only partially ( $0 < a < 1$ ), are near-rational. When  $\pi$  is zero the losses of such producers is zero, as can be seen by the fact that when  $\pi$  is zero,  $z$  is 1. Thus according to (9) the losses from being near-rational when  $z$  is zero will also be zero. These losses will also continue to be small at low levels of inflation, near zero, since the derivative of (9) with respect to  $\pi$  is also zero when  $\pi$  is zero.

Secondly, (9) serves as the springboard for the completion of the model we will estimate below, which is based explicitly on the losses that are entailed from near-rational behavior. To complete the model it is assumed that firm wage and price setters are willing to tolerate losses relative to their profits, only up to a given threshold,  $\epsilon$ , before they will switch to fully rational behavior. We assume that these thresholds are normally distributed with mean  $\mu_\epsilon$  and standard deviation  $\sigma_\epsilon$ . The fraction of near-rational price setters accordingly will then be:

$$(10) \quad 1 - \Phi \left[ \frac{1 - \left[ \frac{\alpha z}{z^\alpha - 1 + \alpha} \right]^{-\beta+1} - \mu_\epsilon}{\sigma_\epsilon} \right]$$

where  $\Phi$  is the standard cumulative normal distribution, and  $\mu_\epsilon$  and  $\sigma_\epsilon$  are respectively the mean and standard deviation of the distribution of the thresholds  $\epsilon$ .

Finally, (9) also yields benchmark estimates of the size of losses because of near-rational behavior. Table 1 shows the fraction of the profits of the fully rational firm sacrificed by the near-rational firm at different rates of inflation for two different values of  $a$  and two different values of both  $\alpha$  and  $\beta$ .

<b>Table 1</b>								
Percent of the Profits of a Fully Rational Firm Lost by Near-Rational Behavior in the Treatment of Inflation								
Inflation Rate	a = 0 (Near-rational firms ignore inflation)				a = .7 (Near-rational firms weight inflation)			
	Elasticity of Demand ( $\beta$ )				Elasticity of Demand ( $\beta$ )			
	3		10		3		10	
	$\alpha=.1$	$\alpha=.75$	$\alpha=.1$	$\alpha=.75$	$\alpha=.1$	$\alpha=.75$	$\alpha=.1$	$\alpha=.75$
1%	.009%	.002%	.04%	.01%	.001%	.000%	.004%	.001%
2%	.04%	.01%	.16%	.04%	.003%	.001%	.01%	.004%
3%	.08%	.02%	.36%	.10%	.007%	.002%	.03%	.01%
4%	.14%	.04%	.64%	.18%	.01%	.003%	.05%	.02%
5%	.22%	.06%	1.00%	.27%	.02%	.005%	.08%	.02%
7%	.43%	.12%	1.92%	.53%	.04%	.01%	.16%	.04%
10%	.87%	.24%	3.84%	1.06%	.07%	.02%	.31%	.09%

To put the values in table 1 in perspective, consider the findings of Leonard (1987) and Davis *et. al.* (1996) that the typical firm annually experiences shocks to demand that cause it to adjust its size up or down by roughly 10%. Failing to adjust capacity to accommodate such a shock would cost a firm 10% of its profits. Thus it does not seem hard to believe that for the typical firm, the issue of how to treat inflation in setting prices is far down the list of items demanding managerial attention—at least with inflation under 5%.

*Implications for the Long-Run and the Short-Run Phillips Curve*

The model also allows easy derivation of both a short-run Phillips Curve with given expectations of price inflation and a long-run Phillips curve where actual and expected inflation must coincide.

The short-run wage-Phillips Curve is obtained from wage-setting behavior and the equation for the average wage. The average wage in this economy will be:

$$(11) \quad \bar{w} = \Phi w_r + (1 - \Phi) w_{nr}.$$

Using the wage setting behavior of the rational and near-rational firms,

$$(12) \quad \bar{w} = \Phi \left[ \frac{A-Cu}{B(1-\alpha)} \right]^{1/\alpha} w_r^R + (1 - \Phi) \left[ \frac{A-Cu}{B(1-\alpha)} \right]^{1/\alpha} w_{nr}^R$$

which can be rewritten as,

$$(13) \quad \bar{w} = \Phi \left[ \frac{A-Cu}{B(1-\alpha)} \right]^{1/\alpha} w_{-1} (1 + \pi^e) + (1 - \Phi) \left[ \frac{A-Cu}{B(1-\alpha)} \right]^{1/\alpha} w_{-1} (1 + a\pi^e)$$

using the definition of the reference wage. Dividing the left hand and the right hand side by  $\bar{w}_{-1}$  and collecting terms yields the relation:

$$(14) \quad (1 + \pi_w) = \left[ \frac{A - Cu}{B(1 - \alpha)} \right]^{1/\alpha} [1 + \Phi \pi^e + (1 - \Phi)a\pi^e],$$

where  $\pi_w$  is the rate of wage inflation. Taking the logs of both the right hand side and the left hand side of (14), approximating  $\ln(1 + \pi_w)$  by  $\pi_w$ ,  $\ln[1 + \Phi \pi^e + (1 - \Phi)a \pi^e]$  by  $[\Phi + (1 - \Phi)a] \pi^e$ , and  $\ln[A - Cu]/[B(1 - \alpha)]^{1/\alpha}$  by its linear approximation,  $d - e u$ , yields the short-run wage-Phillips Curve:

$$(15) \quad \pi_w = d - e u + (1 - f) \pi^e,$$

where  $f = (1 - a)(1 - \Phi)$ .

A price Phillips Curve, which is similar to (15), can also be derived from the model. The slight difference between the price Phillips Curve implied by our model and the wage Phillips Curve (15) is the presence of a change in unemployment term in the price Phillips Curve. This term enters because changes in the unemployment rate will cause changes in productivity and hence, via (6), in the price/wage markups.<sup>7</sup> We take this into account when we estimate the model by allowing lags on the unemployment rate. The steady state Phillips Curves with constant unemployment will be unaffected by varying markups caused by varying unemployment.

The short-run Phillips Curve (15) should come as no surprise. If all inflation had been included in the mental frames of the firms, which are setting wages and prices in this model, then the coefficient  $f$  would be equal to zero. The near-rational firms, which constitute a fraction  $1 - \Phi$  of all of the firms, ignore a fraction  $(1 - a)$  of inflation. As a consequence, the Phillips Curve (15) mimics the usual inflation-augmented Phillips Curve, but with a fraction  $(1 - a)(1 - \Phi)$  of the expected inflation ignored. Thus the Phillips Curve of the form (15) is not just an artifact of our illustrative model of price and wage setting. As long as a fraction of inflation is ignored or underweighted in near-rational wage and price setting, that fraction of inflation should fail to enter the inflation augmentation term. A whole spectrum of other models in which various combinations

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<sup>7</sup>The price Phillips Curve will be of the form:

$$\pi = c - e u + f \pi^e + (1 - f) h \Delta u^e,$$

where  $h = -C/[b(1 - \alpha)]$ ,  $u$  is current unemployment, and  $\Delta u^e$  is the expected change in unemployment.

of firms and workers are ignoring or underweighting inflation in their mental frames will yield similar results.

Using (15), the long-term Phillips Curve—where actual and expected inflation are equal—will be:

$$(16) \quad u^n - u = \frac{1}{e} f \pi.$$

where  $u^n$  is the natural rate of unemployment if all firms are rational. Its value in this model is  $d/e$ .

The Phillips Curve (16) will be bowed out and then forward bending. At zero inflation  $\pi$  is zero and therefore unemployment is at the natural rate. At very high inflation all firms will have given up being near-rational. The losses from near-rational behavior will be sufficiently large that by (10),  $\Phi$  will be close to one—so that  $f$ , which is  $(1 - \Phi)(1 - a)$ , will be close to zero. Thus at both very high and very low inflation unemployment will be close to the natural rate, which is the level of unemployment that would occur if all firms were totally rational. At inflation above zero, unemployment will always be below the natural rate since  $f$  will always be positive.

Figure 1 portrays the rate of unemployment that corresponds to different levels of inflation in the long run with bench-mark parameters. We have assumed that near-rational firms completely ignore inflation ( $a=0$ ). We chose the parameters describing the distribution of  $\Phi$  so that at least  $1/2$  of all firms are always fully rational ( thus  $\mu_\epsilon$  is zero), and 95 percent of all firms are rational by the time inflation is 5 percent (which implied a value for  $\sigma_\epsilon$  of .002 or .2% of normal profits). We

also chose  $\alpha$  at .1 and an elasticity of demand ( $\beta$ ) of four, though as we will discuss below, these assumptions hardly matter at all for the shape of figure 1.

The optimal rate of inflation is the level that maximizes the product of  $f$  and  $\pi$ . This level of inflation, according to (16) will minimize unemployment. For the parameter values chosen to create figure 1 that inflation rate is 2.6%. At that rate of inflation the long-run equilibrium rate of unemployment is 1.7 percentage points lower than at either a rate of inflation of zero or a rate above 6 percent.<sup>8</sup>

**[Figure 1 about here]**

Why does employment rise with inflation at low rates of inflation? In our model, inflation is not underestimated, but instead it is under weighted in the reference wage used for wage setting. This has the same consequences as underestimation. Near-rational firms either ignore or fail to fully project inflation so they set lower wages, and therefore also set lower prices, relative to nominal demand, than they would if they were fully rational. At these lower prices both output and employment will be higher. These higher levels would also occur in the slightly different version of the model in which workers' underestimate the impact of inflation.

In our model the level of inflation that yields the minimum obtainable unemployment rate

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<sup>8</sup>Interestingly, our choices of the values of the elasticity of demand ( $\beta$ ), and the curvature of the productivity function ( $\alpha$ ), hardly matter for the shape of the curve in figure 1 or for the optimal rate of inflation and unemployment. Once we set the fraction of firms that are near-rational at two points we have described the curve for a given value of  $a$ . This result reflects a finding that will surface again later when we estimate the model, which is discussed in more detail in the next section—the loss function is very nearly approximated by a constant times the square of inflation so that the argument of the cumulative normal in our model can be very well approximated with two parameters.

will be the optimum. Since the firms are monopolistic competitors, producing more output increases the welfare of the owners of the firms. Also, with the labor market characterized by the payment of efficiency wages, unemployed workers are happy to supply more labor if it is demanded. Their welfare will be improved if they obtain work at the going wage, so workers, as well as owners of firms, will have increased welfare as employment increases. Our concept of the optimal rate of inflation ignores both the transactions (*e.g.* the so-called “shoe-leather”) costs of higher inflation as well as the tax-distortion effects, both of which we consider to be small.<sup>9</sup> It also ignores other considerations, such as inflation's redistributive effects, loss of confidence in the currency, effects on exchange rates, and the improved allocation of resources that results from small amounts of inflation in the presence of nominal wage and price rigidities. We continue to refer to the rate of inflation that minimizes the unemployment rate as the optimal rate below despite the uncertainty about what rate would be optimal in the broader context that included these considerations.

### **Empirical Evidence for Near-Rational Wage and Price Setting**

In this section we discuss three related types of evidence for the importance of the type of behavior we describe. We begin with a recounting of the findings of Brainard and Perry's recent analysis of a Phillips Curve model with time-varying parameters. We then do a simple exercise in

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<sup>9</sup>Feldstein (1997) has estimated very large deadweight losses from the tax distortions of going from zero to two percent inflation. His calculations omitted the tax sheltering of pension plans, 401k's, IRA's and other tax-saving devices. The deadweight losses will be almost zero for savers who fail to exhaust their possibilities for tax deferred savings. Kusko, Poterba, and Wilcox (1994) found only 1 percent of 401k participants in a medium-sized manufacturing plant were constrained, and, similarly, Papke (1995) found that less than 1 percent of contributions to 401(k) plans were in excess of \$5,000 in 1987. Feldstein's calculations were based on a model with no uncertainty-induced precautionary savings. Independent of any considerations of tax sheltering, inclusion of such precautionary saving will likely reduce by almost 90% the estimates of the tax-distortion welfare loss.

which we estimate Phillips Curves on a split sample to see how the estimated coefficient of inflation differs between periods of high and low inflation. Finally, we estimate the model described in the previous section and present estimates of the optimal rate of inflation and the gains from being at the optimal rate as opposed to higher or lower rates of inflation.

### *Time Varying Parameters*

In the Brainard and Perry paper that we described at the outset, the authors were addressing how uncertainty affects policy making. Their empirical work demonstrating one key source of uncertainty reveals precisely the departures from conventional NAIRU models that our model predicts. Previous work examining how NAIRU had varied over time assumed the NAIRU framework and allowed time variation only in the intercept of the equation.<sup>10</sup> Brainard and Perry applied a general Kalman filter estimation that permits all the key Phillips curve parameters to vary—lagged inflation and unemployment as well as the intercept.—and lets the data to choose the allocation of time variation among them.<sup>11</sup> Figure 2, which summarizes their results with CPI inflation as the dependent variable, shows substantial time variation in the coefficient of the lagged inflation term and virtual stability in the intercept and the inverse unemployment rate, which they measure by the unemployment rate of 25-to-54 year old men to account for demographic changes over time. The coefficient on lagged inflation is low during periods of low inflation and approaches 1.0 only in the inflationary middle years of the period.

**Figure 2 about here**

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<sup>10</sup>For typical applications see James H. Stock and Mark W. Watson (1998) and Robert J. Gordon (1998).

<sup>11</sup> See Brainard and Perry (2000).

The virtual stability over time in the unemployment coefficient and intercept in the Brainard-Perry time-varying estimates is also worth noting. Rather than attributing the episodes of sustained low unemployment to declines in a NAIRU that is invariant to inflation, these results attribute them instead to a change in price and wage setting behavior that accompanied periods of low inflation. The juxtaposition of coefficients on lagged inflation that change with the inflation regime with constant coefficients elsewhere is predicted by the model we have described above.

Brainard and Perry compared their Kalman filter estimates with recursive least squares estimates, which are also shown in figure 2. These comparisons suggest why conventional estimation has seemed to support the NAIRU model since it was first introduced in the inflationary mid-1970s by Modigliani and Papademos (1975). Before that time, lagged inflation in Phillips curves was consistently estimated to have a coefficient well below 1.0. But the large increase in inflation in the mid-1970s corresponded to the period of large variance in inflation and fixed coefficient estimation has been dominated by that episode ever since. If the coefficients in fact have varied over time, any procedure that assumes that they are fixed will yield misleading results. This includes the recursive estimates which treat them as fixed in each interval over which they are estimated.

#### *Periods of Low and High Inflation*

The postwar U.S. economy has experienced extended episodes of both low and moderately high inflation that permit direct comparison of the NAIRU model with our model. Conventional NAIRU models use a modified Phillips curve in which lagged inflation is taken as a measure of adaptive inflationary expectations and the coefficients on lagged inflation sum to 1.0. By

contrast, our model allows the possibility that the coefficient on expected inflation will be lower in extended periods of low inflation than in extended periods of high inflation. Absent estimation biases, we would expect the coefficient to approach 1.0 in a sufficiently inflationary environment. We first look at the empirical evidence using the conventional adaptive expectations framework. We then provide evidence using direct measures of inflationary expectations that address Sargent's [1971] criticism of the assumption that the coefficient on lagged inflation must equal one in an accelerationist model. Sargent argued that a coefficient of less than one on lagged inflation may not reflect incomplete projection of inflation but rather forecasters' views that the process generating inflation does not have a unit root. By using direct measures of inflationary expectations we can rule out the possibility that our results reflect differences in how people form expectations rather than how they use them.<sup>12</sup>

In order to separately estimate wage and price Phillips curves for periods of low and high inflation, we sorted the quarters since the Korean War according to the average CPI inflation rate in the five-year period ending each quarter. We first classified quarters with average inflation rates below 3 percent as low inflation and quarters with average inflation rates above 4 percent as high inflation.<sup>13</sup> By this sorting, the low inflation quarters run from 1954:1 through 1969:1 and from 1995:3 through 1999:4, the end of our sample period. The high inflation quarters run from 1970:2 through 1986:1 and from 1990:4 through 1993:2. There are 77 quarters in the high inflation sample and 77 quarters in the low inflation sample. The mean CPI inflation rates in the two samples are 2.0 percent and 6.3 percent. This separation was used in half the wage and price

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<sup>12</sup>We are grateful to a seminar participant at the Bank of Canada for suggesting this approach.

<sup>13</sup>By sorting our sample on the basis of long lags of the endogenous variable we considerably reduce concern about sample selection on the basis of an endogenous variable.

inflation regressions. In the other half we limited the low inflation sample to quarters with inflation rates below 2.5 percent, which brought the sample size down to 62 quarters and reduced the mean CPI inflation rate in the low inflation sample to 1.9 percent.

### *Estimates with Adaptive Expectations*

The quarterly Phillips curve equations we estimated were intended to span the specifications that analysts have used in conventional estimation of NAIRU models except for the fact that we did not constrain the coefficients on lagged inflation. To this end, we tried a large number of data combinations and specifications on both wage and price Phillips curves, and ran each separately for the low and high inflation samples just described. In all cases the dependent variable was an annualized inflation rate in either wages or prices, and the explanatory variables were current or lagged values of unemployment, price inflation and, for the wage equations, trend productivity growth. For price inflation we used the CPI, the GDP deflator and the PCE deflator and estimated price Phillips curves with each. Twelve values of lagged inflation were used as explanatory variables. For wage inflation we used the best series available for any time period, linking private ECI wages and salaries for 1980-1999 to the adjusted hourly earnings index for the nonfarm economy for 1961-1980 and to adjusted hourly earnings in manufacturing for 1954-1961. Twelve lagged values of CPI inflation were used as explanatory variables. For unemployment we used the total rate, the 25-to-54 year old male rate, and Robert Shimer's demographically adjusted series. We used the current and three lagged values of unemployment and, alternatively, the current and eleven lagged values. For the wage Phillips curves, we used two estimates of trend productivity growth, one being the series created by Robert Gordon and the

other a smoothed version of that series. We ran regressions with the productivity coefficient both freely estimated and constrained to be 1.0 (for the wage inflation equations), and with just the current trend and with the current plus seven lagged values of the trend.<sup>14</sup>

### **Figures 3 and 4 about here**

The key results are summarized in figure 3 for equations explaining wages and in figure 4 for equations explaining prices. The figures present the results of 144 and 72 specifications respectively. Each point represents the sum of the coefficients on lagged inflation estimated for the low and high inflation samples for one specification. If the sum of coefficients were similar for the two samples, the points would cluster along the forty-five degree line. If they were similar and near 1.0, the points would cluster near the upper right corner. In fact, for both wages and prices, and over the wide range of specifications and data we used, the points cluster near 1.0 on the high inflation axis, but on the low-inflation axis, they range from around zero to around 0.5 for the wage equations. This is consistent with the predictions of our model. The range on the price equations is broader and less conclusive. The third of the observations at the highest end of the range are from equations using the PCE deflator. The mean values of the coefficients on the high and low inflation axes respectively are 0.25 and 0.82 for the wage equations and 0.60 and 0.95 for the price equations.

#### *Direct Measures of Inflationary Expectations*

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<sup>14</sup>All equations also used the customary dummy variables for the guidepost period in the 1960s and the price control period of the 1970s, and used the difference between inflation with and without oil prices in 1979-1980 as an additional variable.

As in Brainard and Perry (2000) the results just described cast doubt on conventional estimates with the NAIRU model. However they both treat expectations as adaptive and so cannot refute Sargent's (1971) criticism that rational expectations are formed differently and that the coefficient on properly measured expectations might be 1.0. We now address this issue by using direct measures of expected inflation as explanatory variables in place of distributed lags of actual inflation rates, while maintaining our division of the sample into periods of high and low inflation. The other explanatory variables are the same as those used in the regressions behind figures 3 and 4. We used the two direct measures of expected rates of inflation that are available over our sample period: one from the Survey of Consumer Finances and the other the Federal Reserve's Livingston Surveys. Figures 5 and 6 plot the estimated coefficients on expected inflation for the variously specified wage and price regressions respectively. As with the results using adaptive expectations, the coefficients on expected inflation are substantially different in the low- and high-inflation periods. For 288 wage equations the low- and high-period means are 0.29 and 0.85. For 144 price equations the means are 0.25 and 1.00.

**Figures 5 and 6 about here**

These results support our general hypothesis even more convincingly than the results with adaptive expectations. Not only do they address the point that the relevant coefficient for natural rate theory is not necessarily the coefficient estimated with adaptive expectations, but the results are as clear about price inflation as they are about wage inflation.

One possible objection to the results presented here and in the next section is that the

lower coefficients on inflationary expectations during periods of low inflation are an artifact of measurement error. For example, if the variance of measurement error is constant while the variance of true inflationary expectations are higher in times of high inflation, then the coefficient on expectations could be biased towards zero more in times of low inflation than high inflation. We investigated this possibility. While it is true that the variance of expectations is higher in periods of high inflation, it is also true that the sampling error in both the SCF and the Livingston surveys are also higher. In fact, the sampling error is so much higher that the computed bias is *higher* in the low inflation periods imparting a bias *against* our finding that the coefficient on expectations is lower in periods of low inflation. Sampling error may not be the only source of error in the survey expectations. Neither survey may be asking the right people with the right weights. In an attempt to approximate how much error this problem might introduce we computed the bias that would be caused if the measurement error variance in expectations was equal to the variance of the residual of a regression of one of our survey expectations on the other. Again we found that the “measurement error” variance grew faster than the conditional variance of the expectations so that the bias caused would work against our finding that the coefficient on expectations was lower when inflation was low.

### **Estimating The Model**

Previously we showed how a Phillips Curve type relation can be derived from our theoretical model (equation 15). In this section we present estimates of the model and of the optimal rate of inflation and the gain in employment that is possible from moving to the optimal rate. This section will first discuss the specification of the model we estimate, then our benchmark

results, and finally, an analysis of their robustness.

### *Specifications*

In theory, with a large enough sample, it would be possible to estimate the full model presented above. The elasticity of demand ( $\beta$ ), the parameter for the curvature of the unit cost function ( $\alpha$ ), and the parameters of the distribution of rationality thresholds ( $\mu$  and  $\sigma$ ), all have different effects on the objective function. However, in practice, it was impossible to estimate more than the mean of the distribution of rationality thresholds, and one of the other parameters because all three of them—the elasticity of demand, the curvature of the unit cost function and the standard deviation of the distribution of rationality thresholds—act in much the same way to determine the impact of past rates of inflation on the cumulative normal term. (See equation (15) above).

The lack of identification in practice can be understood if we consider a Taylor series approximation to the argument of the cumulative normal in equation (15) expanded around a value of zero inflation. There is no reason to expect that the argument will be exactly zero at zero inflation so the constant term will likely be present. As we have shown above, the first derivative of the firm's loss function with respect to inflation is zero at zero inflation and very small at most rates of inflation less than 10 percent. Thus the first order term of the Taylor series expansion of the argument of the cumulative normal will also be zero. Second and higher order terms will be present, but analysis we have conducted of the loss function suggests that with inflation between zero and ten percent, with elasticity of demand between 2 and 10, with curvature of the unit cost function from .05 to .95, and any value of the standard deviation of the distribution of rationality

thresholds, the third order and higher terms are unimportant. An approximation of the loss function of the form  $E \pi^2$ , where  $E$  was chosen so that the approximation was exactly equal to the loss at 5% inflation, was never off by more than 3% of the loss. One parameter is all that is necessary to capture the effects of all three parameters from the model ( $\alpha$ ,  $\beta$ , and  $\sigma$ ) on the derivative of the argument of the cumulative normal with respect to inflation.

We thus estimate a Phillips curve of the form:

$$(17) \quad \pi = d + \Phi(D + E \pi_L^2) \pi^e - e u + g X + \epsilon$$

where  $\pi$  is the rate of inflation,  $\Phi$  is the cumulative standard normal density function,  $\pi^e$  is inflationary expectations,  $u$  is a term capturing the effects of current and lagged unemployment on inflation,  $X$  is a matrix of dummy variables for oil shocks and price controls,  $\epsilon$  is the error term, and  $d$ ,  $D$ ,  $E$ ,  $e$  and  $g$  are parameters to be estimated.<sup>15</sup>

The term  $\pi_L$  represents the effects of past inflation on the likelihood that people will act rationally towards inflation. Our theory tells us nothing about the way in which inflation should matter other than the sign of  $E$ , so we proxy  $\pi_L$  with several different parsimonious specifications. The first is a geometrically declining weighted moving average of past values of inflation:

$$(18) \quad \pi_L = (1 - \delta) \pi_{L-1} + \delta \pi_{-1},$$

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<sup>15</sup>This specification ignores the parameter “a” from the theoretical model. In theory that parameter could be estimated, but we do not take the theoretical model that literally. Instead we imagine that there is a continuum of reactions to increasing inflation with people putting more and more weight on it until their behavior resembles that of the rational economic actor in the standard model. The model we estimate here can be thought of as a model where a fraction  $(1-\Phi)$  of people are ignoring inflation, or the phi function can be thought of as approximating a more general function that reflects how much weight the average person is putting on inflation in making economic decisions.

where  $\delta$  is a parameter to be estimated.

Alternatively we estimate  $\pi_L$  as

$$(19) \quad \pi_L = \frac{\sum_{\substack{i=1 \\ i\lambda < 1}}^{24} (1-i\lambda) \pi_{-i}}{\sum_{\substack{i=1 \\ i\lambda < 1}}^{24} (1-i\lambda)}$$

where the parameter  $\lambda$  is estimated. Our final two specifications for  $\pi_L$  treat it as a 4-year moving average of past inflation with equal weights, or with the relative weights of quarters from each year are estimated (three additional parameters).

It is standard practice to proxy inflationary expectations with lagged values of inflation in Phillips Curve estimation. In many specifications discussed below we follow that tradition. When we do, we use either a 12 quarter unrestricted lag or one of the methods used to construct  $\pi_L$  to construct  $\pi^e$ . However, we also want to rule out the possibility that changes in the coefficient on  $\pi^e$  might reflect changes in the process by which expectations are formed rather than how they are used. Thus we also use direct survey measures of inflationary expectations for  $\pi^e$  in some specifications.

Our different specifications include several different measures of unemployment and also different numbers of lags. The unemployment term,  $u$ , is constructed using one of three data series. The first is the aggregate U.S. unemployment rate from the Current Population Survey. Because this variable may be influenced by changing demographics, we have also considered two

alternative measures: the unemployment rate for prime age males and Shimer's demographically corrected series. (See Shimer, 1998). We also vary the number of unemployment lags from zero to 11 quarters.

For the dependent variable we variously use four different measures of inflation: the annualized percent change in the consumer price index (CPI-UXG), the gross domestic product deflator, the personal consumption expenditures deflator, and the index of wage and salary compensation constructed by Brainard and Perry (2000). When we use the percent change in the compensation index as the dependent variable we subtract off a measure of trend productivity growth. The three specifications of this trend are: A measure based on Gordon (1998), the measure we constructed for our 1996 paper, and a 16-quarter moving average.

Since the form of the Phillips curve here is similar in some respects to the one in our previous paper (Akerlof, Dickens and Perry (1996)) that modelled the implications of downward nominal wage rigidity, we also examine the question of whether we can successfully estimate a Phillips curve which embodies the insights from that model as well as the current one. Below we estimate a number of specifications that augment equation (17) with the term for nominal rigidity from that previous paper.<sup>16</sup> When we nest that model we must also estimate its key parameter—the standard deviation of desired wage changes along with the other parameters from the current model. (See Akerlof, Dickens and Perry (1996, Appendix A) for its specification.)<sup>17</sup>

The model was estimated with quarterly US data from the first quarter of 1954 through the

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<sup>16</sup>The inclusion of the term for nominal rigidity could be motivated if we included firm profitability or firm specific labor market considerations into the productivity function. That would produce heterogeneity in desired wage setting with firms constrained by the floor of no nominal wage decrease forced to pay a higher wage as in the model in our previous paper.

<sup>17</sup>We leave out the term for change in profits, which could not be robustly estimated.

last quarter of 1999, though we vary the end date in some specifications to check the extent to which our results depend on the experience of the 1990s. Data sources and the specification of the dummy variables for price controls and oil shocks can be found in the appendix.<sup>18</sup> All the parameters of the model were estimated simultaneously by non-linear least squares.

### *Results*

Table 2 presents results for four different estimates with five types of variation: in the dependent variable, in the method of constructing  $\pi^e$  and  $\pi_L$ , in the unemployment measure and its lags, in the sample period and in the inclusion of the term for nominal rigidity.

<b>Table 2</b> <b>Estimated Parameters for Near Rational Phillips Curve</b> (standard errors in parenthesis)				
Independent Variables and Characteristics	Dependent Variable			
	CPI	GDP deflator	PCE deflator	Compensation Index-prod. growth
Constant	.042 (.009)	.028 (.008)	.024 (.011)	.017 (.003)
Unemployment	-.54 (.12)	-.45 (.12)	-.40 (.16)	-.39 (.07)
D (Constant in coefficient on expectations)	-.70 (.39)	-.88 (.45)	-.23 (.47)	-.32 (.22)
E (Coefficient of $\pi_L^2$ in coef on expectations)	601 (180)	2824 (1119)	1210 (552)	1311 (355)
Standard deviation of desired wage change from term for nominal rigidity	term not included	term not included	.020 (.013)	term not included
Method for constructing $\pi_L$	geometric	16q MA (equal	geometric	linear

<sup>18</sup>We use dummy variables rather than an import price or energy price measure because we believe that these were atypical events that had atypical effects on the economy.

		weights)		
Method for constructing $\pi^e$	SCF	12 unrestricted lags	geometric	Livingston
Unemployment measure and number of lags	Total 0 lags	Total 11 lags	Shimer 7 lags	Male 3 lags
Sample Period	54:1-89:4	54:1-99:4	54:1-99:4	54:1-99:4
Natural Rate	7.7	6.4	6.1	4.3
Optimal Rate of Inflation	3.2	1.6	2.3	2.0
Lowest Sustainable Rate of Unemployment	4.6	4.4	4.6	2.2
Durbin-Watson Statistic	1.4	2.0	1.9	1.1
R <sup>2</sup>	.792	.698	.707	.764

Our first focus of attention is the estimated value of the cumulative normal multiplying inflationary expectations when inflation is zero. In the theoretical model this corresponds to the fraction of firms behaving in a fully rational fashion at zero inflation. The model predicts that this fraction will be less than unity, and also that as inflation increases above zero, the fraction of rational firms will rise. Both of these predictions yield tests of the model.

The NAIRU specification for the Phillips curve is nested in our model and can be obtained if the value of  $D$  is sufficiently high. For example, if  $D$  were 2 or higher the coefficient on inflationary expectations would never fall below .97 and there would be little room for changing experience with inflation to affect the coefficient on inflationary expectations. All of the four estimated values of  $D$  imply coefficients on expected inflation less than .5 at zero inflation. The lowest implies a coefficient of .19. In all four cases a value of  $D$  which would imply a coefficient of .9 or greater (1.28) can be rejected at conventional levels of significance.

The instantaneous effect of increasing inflation above zero can be computed as one minus

the cumulative normal evaluated at  $D$  divided by the sum of the coefficients on unemployment and its lags. Those values are about -1.5 or larger (in absolute value) in the specifications presented here. Thus to a first-order approximation raising inflation from zero to one percent will cause a reduction in unemployment of 1.5 percentage points or more.

The term which most distinguishes our model from that of the textbooks is the coefficient of the square of lagged inflation in the cumulative normal multiplying inflationary expectations ( $E$ ). If  $E$  is zero, the coefficient on expectations will not vary with past rates of inflation. Our theory says it should and that is what we find in each of the specifications we have estimated. In all four specifications presented above  $E$  is large, and more than twice its estimated standard error. Going from zero to five percent inflation would increase the argument of the cumulative normal by 1.5 to 7.1 depending on the specification. Except with CPI inflation as the dependent variable, the coefficient on inflationary expectations is above .95 by the time inflation has reached 4 percent. For the CPI-specification the coefficient is .6 at 4 percent inflation and rises above .95 at about 6.5 percent.

Besides allowing us to estimate the effect of inflation on the use of inflationary expectations, estimating our model also allows us to calculate an optimal rate of inflation and the potential employment gains of moving to that optimum. We have computed the optimal rate of inflation for the four models in table 2 from the estimated parameters numerically. We have also computed the natural rate in each model and the Lowest Sustainable Rate of Unemployment or LSRU—the unemployment rate at the optimal rate of inflation. The optimal rate of inflation ranges from 1.6 percent to 3.2 percent. The difference between the natural rate and the LSRU ranges from 1.5 to 3.1 percentage points. Figures 7 a,b,c, and d show the long-run relationship

between inflation and unemployment implied by each of the four specifications estimated in table 2.

The values of the coefficient of inflationary expectations implied by our parameter estimates is plotted in figures 8a,b,c, and d for each of our four specifications. In all cases coefficient values vary considerably over the sample. In all four specifications the coefficient on inflation reaches a maximum value of one for at least a year at some point during the sample period in the early to mid 80s. The four specifications differ in the exact timing of the increase in the 70s, in how the 50s and 90s are treated, and in the date of the end of the period of a coefficient of one on inflation.

These figures can be compared to the time path Brainard and Perry estimated for the coefficient on inflation. Our estimates imply considerably more abrupt changes and more persistence. They also imply more variation. However, it must be remembered that the method Brainard and Perry used to estimate their values for the coefficient on inflation imposes smoothness on the changes. When we smooth our estimates (not shown) they begin to resemble the time path that Brainard and Perry found with one major difference. The Brainard Perry estimates peak earlier and fall off more abruptly than our smoothed estimates.

We have varied the specifications presented above to anticipate possible objections to our results. The specification with the CPI as the dependent variable shows that our results do not depend on the experience of the 90s which may be atypical. Since non-linear estimation is difficult when many parameters are being estimated, we have generally used very parsimonious specifications for the lags on past price inflation when constructing inflationary expectations. One might object that this parsimony forces the coefficient on inflation to do the work that a richer lag

structure would do. The specification where the GDP deflator is the dependent variable answers this by matching the richest possible lag structure for price expectations (12 quarter unrestricted) with the most parsimonious specification of the term in the coefficient of expectations. Likewise, in most specifications, including lagged unemployment, and/or our term for nominal rigidity does not change our fundamental results.

Our Durbin-Watson statistics for the two specifications using survey expectations show considerable serial correlation. We have not attempted to correct for this problem because we lack a credible instrument for price expectations which are endogenous with respect to the error in the Phillips curve. We are unhappy with this drawback of the analysis, but estimates of our model we have tried using simulated data suggest that the bias from ignoring the serial correlation in the parameters we care about is minor.

### *Robustness of Results*

As we have noted above, there are many aspects of the specification that are not dictated by the theory. Our approach to this problem has been to estimate a wide array of different specifications to determine whether our primary results are sensitive to changes in the specification.

Because both the estimation of the model and the numerical analysis of the results currently require human intervention, we have not been able to mechanize the process of sensitivity testing. Thus we have not been able to do an exhaustive specification search. Instead we estimated 218 different specifications. Many were run to test specific concerns. However, most were chosen randomly.<sup>19</sup> Our survey of the results of these specifications yields the

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<sup>19</sup>We set a goal of 200 specifications, met that goal, and then estimated a few more to check specific concerns that arose in the process of evaluating the 200 specifications. In randomly

following generalizations:

**Figure 9 about here**

1) Most important, nearly all the point estimates imply that significant gains in employment are possible by increasing inflation from zero to a rate above 1.5 percent. This can be seen in figure 9 which plots for each specification the optimal rate of inflation and the reduction in unemployment that obtains from increasing inflation from zero to the optimal rate. There were only 12 specifications where the estimated gain was less than 1 percentage point and only one where it was negligible. This specification was a wage Phillips curve with a rich lag structure for price expectations while the inflation term in the coefficient on expectations was constrained to be an equally weighted sixteen quarter moving average of past inflation. Allowing a richer specification for the impact of inflation on the use of expectations eliminates this result. Of the other eleven specifications where the estimated impact is less than one percent, all are at least a half a percentage point. Most of the specifications are wage equations, and none use the PCE deflator as the dependent variable. Only one uses survey expectations. In no case are the parameters of the inflation coefficient very precisely estimated so that values more typical of other specifications cannot be ruled out.

2). It is not possible to robustly identify the relative importance of the effects of nominal rigidity vs. the effects of near rationality. The majority of specifications that included our term for

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choosing specifications we allowed all options with equal probability except that we found that the 12 quarter unrestricted lag on inflation for the price expectations term was always computationally burdensome so we did not include those specifications in those that were randomly chosen.

the effects of nominal rigidity give results like those for the PCE in table 2. These do suggest a role for both nominal rigidity and near rationality. However, in many specifications that include both effects, the effect of past inflation on the coefficient of expectations is not measured precisely being about the same size as its estimated standard error.<sup>20</sup> In other cases, the optimization routine was trying to drive estimates of the standard deviation of desired wage changes to zero. In six specifications not represented in figure 9 we obtained converged estimates for the parameters, but the estimated values for sigma were sufficiently large that there was no single rate of inflation at which the unemployment rate was minimized. It simply fell to the natural rate asymptotically as in the models estimated for our 1996 paper.

3) We encountered few problems with applying non-linear estimation. We did look for and find a few cases where there were multiple local minimums, but these reflected minor differences in the lag structures that were not substantive. Of the 218 specifications we estimated we were unable to obtain converged values for about 30. This might be a serious concern because under the hypothesis of fully rational behavior the model's parameters are not identified and it might be that the non-linear estimation program is trying to drive the constant term in the coefficient on inflationary expectations to infinity in order to drive the coefficient on expectations to 1.<sup>21</sup> However, this is not what was happening in any of the cases of convergence problems that we encountered.<sup>22</sup>

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<sup>20</sup>In contrast, when the term for nominal rigidity was not included the coefficient on the square of past inflation was nearly always 1.7 times its estimated standard error or more.

<sup>21</sup>When we generated standard data with a standard Phillips Curve model and attempted to estimate our model on it this is what happened.

<sup>22</sup> Instead we had one of three other problems: 1) the program was trying to drive the sigma to zero, 2) the program was driving the constant term in the coefficient of expectations to *negative* infinity and the coefficient on the square of past inflation to infinity in order to eliminate coefficient values between 1 and the lower floor, or 3) in some very rich specifications the first

Overall, the results from estimating our model support the theory we have laid out. They suggest that the macroeconomic policy should aim for an optimal rate of inflation that is in the range of 1.5 to 4 percent. Either higher or lower rates seem likely to result in lower output and employment.

## **Conclusion**

This paper provides an alternative to natural rate models of unemployment. Natural rate models provide a wonderful economics “just-so” story based on the idea that firms and workers take full account of expected inflation in setting current wages and prices. This behavior produces a unique long run unemployment rate that is consistent with any steady rate of inflation and a short run Phillips curve in which unemployment above or below the natural rate causes inflation to decelerate or accelerate.

Our model of the macro economy rests on behavioral underpinnings that are supported by a range of related evidence including the psychological literature on decision making and perception, direct survey evidence on how people react to inflation, and the advice of compensation professionals. We propose that when inflation is low it is not especially salient, and wage and price setting will respond less than proportionally to expected inflation. At sufficiently high rates of inflation, by contrast, anticipating inflation becomes important and wage and price setting responds fully to expected inflation. This behavioral difference between our model and the natural rate model has significant implications both for estimating the relation between inflation and real activity in the macro economy and for informing the conduct of macroeconomic policy.

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derivatives of a group of unrelated parameters became so close to co-linear that it was impossible to invert the approximation to the Hessian used in the maximization routine.

Our model is supported by the evidence. Estimates of coefficients on expected inflation, whether as conventionally measured by lagged inflation or as measured by direct surveys of expectations, are greater when inflation is high than when it is low. Estimates of our model provide further support. Rather than a natural rate of unemployment that is invariant to the rate of inflation, our model traces out a range of equilibrium unemployment rates associated with different ongoing inflation rates. The optimal unemployment rate is the minimum of this range. The natural unemployment rate is a special case: it is the equilibrium unemployment rate at high inflation rates (and ignoring downward wage rigidity, at zero inflation). It is noticeably above the optimal unemployment rate. The optimal rate of inflation is low, perhaps not far from current values, but not zero. Operating with an inflation rate either higher or lower than the optimal leads to a higher rate of unemployment in the long run.

The distinctive feature of our model is especially important for estimation. In recent years, as low inflation rates have come to be the norm, NAIRUs estimated from the empirical counterpart of the natural rate model have proven to be misleading guides to policy makers and economic analysts. In the mid-1990s, these models typically projected 6 percent as the lowest sustainable unemployment rate, yet real output has grown at a 4-percent annual rate since then and the unemployment rate has fallen to 3.9 percent. The NAIRUs estimated for the early 1960s, the previous period of moderate inflation, also appear unrealistic. When adapted for estimation, the model we have developed should provide more useful estimates of the attainable levels of employment and output to serve as guides for stabilization policy and as anchors to longer run projections.

Not only does our model fit the facts better than NAIRU models, it is also more cogent

theoretically. NAIRU models serve well as what Irving Fisher would call “the first approximation.” They are derived from the assumption that all people behave according to what economists call economic rationality, or else their deviations from that behavior perfectly cancel out. This paper relies, as a first approximation, on exactly such economic thinking. But Irving Fisher also urged economists to make “the Second [and even the Third] Approximation.” With aggregate Phillips Curves such further approximations involve departures from perfectly rational decision-making. The evidence available on the subject suggests that the lay public in setting wages and prices do not have the same model of the economy as economists. Given the complication of their decisions and, for the most part, their lack of training as economists, it would, indeed, be surprising if they did. It is thus highly unlikely that the welter of *interdependent* intuitively-based decisions of a real economy will produce a coefficient of inflationary expectations on wage and price inflation that is always exactly one. This paper has offered a theory for such a departure as price and wage setters under-adjust for inflation when it is not very salient and when the cost of such behavior is low. This theory yields an optimal level of inflation and unemployment. It also fits the facts.

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