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Intermediate Goods and Weak Links: A Theory of Economic Development

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I. Introduction

- Huge income differences across countries — why?
- Old ideas: Leontief (1936) and Hirschman (1958)
Linkages: Intermediate goods are another produced input
⇒ an extra multiplier (like capital).
Complementarity: Multiple weak links in production make development hard.
- A model to make these ideas precise
 - Linkages, complementarity, and superstar effect amplify small distortions.

Literature Review

- Kremer (1993) O-Rings. Ciccone (2002) input chains.
- Empirical work: Hall-Jones, Klenow-Rodriguez, Acemoglu-Johnson-Robinson.
- Models: Romer (1994), Benabou (1996), Blanchard-Kremer (1997), Parente and Prescott (1999), Manuelli-Seshadri (2005), Klenow-Rodriguez (2005), Acemoglu and coauthors.
- Political economy and insitutions
 - Why are allocations distorted? Political economy.
 - Why do distortions lead to large differences? This paper.

Linkages through Intermediate Goods

- Capital multiplier: more $K \rightarrow$ more $Y \rightarrow$ more K , etc.
 - Multiplier is $\frac{1}{1-\alpha} = 3/2$ if $\alpha = 1/3$.
 - Broaden capital: Need $\alpha = 2/3 \Rightarrow$ multiplier = 3.
- The Intermediate Goods Multiplier:
 - Telecommunications, electric power, transportation, education, food supply, health care.
 - Production in each of these sectors depends on productivity in the other sectors = linkages
 - Telecom \leftarrow electricity, educated and healthy workers.
 - Electric generation \leftarrow transportation, telecom, health.
 - Health care \leftarrow telecom, transport, food supply.
- Low productivity in one activity reduces output in the others, which in turn reduces output in the original activity.

A Simple Example

$$Y_t = \bar{A} (K_t^\alpha L_t^{1-\alpha})^{1-\sigma} X_t^\sigma$$

$$K_{t+1} = \bar{s}Y_t + (1 - \delta)K_t$$

$$X_{t+1} = \bar{x}Y_t$$

- Steady State:

$$Y = (\bar{A}\bar{x}^\sigma)^{\frac{1}{1-\sigma}} K^\alpha L^{1-\alpha}$$

$$y \equiv \frac{Y}{L} = \left(\bar{A}\bar{x}^\sigma \left(\frac{\bar{s}}{\delta} \right)^{\alpha(1-\sigma)} \right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$$

- Intermediate goods multiplier (with $\sigma = 1/2$):

$$\frac{1}{1-\sigma} \cdot \frac{1}{1-\alpha} = 2 \cdot 3/2 = 3$$

- Ciccone (2002)

The Role of Complementarity

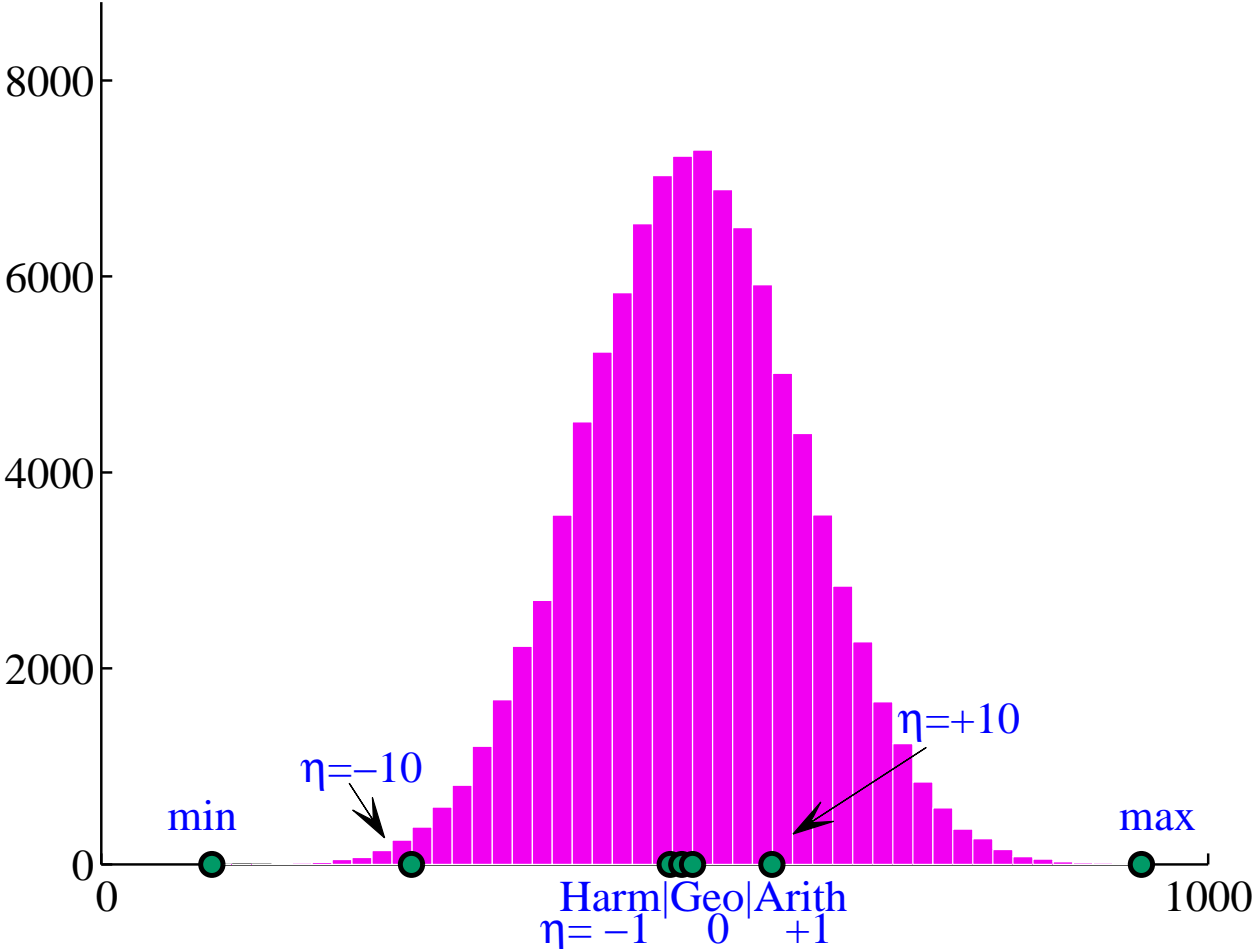
- Linkages and Complementarity often go together (Hirschman 1958)
 - Intermediate goods (energy, raw materials)
- Kremer (1993) O-ring story
 - Production requires 10 things to go right.
 - In the poorest countries, 5 or 6 may be broken.
 - Fixing 1 or 2 will then have small effects.
 - Example: Making socks
- Intuitive channel, but lacking in recent literature

Complementarity and Substitution

$$Y = \left(\int_0^1 z_i^\eta di \right)^{1/\eta}$$

- Degree of complementarity is a parameter: η
 - $\eta = 0$ is Cobb-Douglas (geometric mean)
 - $\eta < 0$ puts more weight on weak links.
 - $\eta > 0$ puts more weight on superstars.
- Generalized (power) mean of the z_i
 - $\eta = 1$ – Arithmetic mean
 - $\eta = -1$ – Harmonic mean
 - $\eta \rightarrow -\infty$ – Minimum (Leontief)
 - $\eta \rightarrow +\infty$ – Maximum (Superstars)
- Power mean is an increasing function of η .

Power Means



II. The Model

The Economic Environment

Production of Variety i :

$$Y_i = A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma} X_i^\sigma$$

Resource constraint (good i):

$$c_i + z_i = Y_i$$

Final uses (substitutes):

$$Y = \left(\int_0^1 c_i^\theta di \right)^{1/\theta}, \quad 0 < \theta < 1$$

Intermediate uses (complements):

$$X = \left(\int_0^1 z_i^\rho di \right)^{1/\rho}, \quad \rho < 0$$

Resource constraint (X):

$$\int_0^1 X_i di \leq X$$

A_i = exogenous productivity, σ = Linkages parameter,
 θ = substitutability of final, ρ = complementarity of intermediates

Environment – continued

Resource constraint (K): $\int_0^1 K_i di \leq K$

Resource constraint (H): $\int_0^1 H_i di \leq H \equiv \bar{h}\bar{L}$

Capital accumulation: $\dot{K} = I - \delta K$

Resource constraint (GDP): $C + I \leq Y$

Preferences: $U = \int_0^{\infty} e^{-\lambda t} u(C_t) dt$

Allocating Resources

- Two ways:
 1. **Symmetric**: A “rule of thumb” allocation, like Solow.
 2. **Competitive Equilibrium**: With tax wedges / distortions.
- Advantages of starting with symmetric
 - Easy to solve for; delivers some key results.
 - Important benchmark for understanding CE.
- DEFINITION: The *symmetric allocation* has $K_i = K$, $H_i = H$, $X_i = X$, $I = \bar{s}Y$, and $z_i = \bar{z}Y_i$, where $0 < \bar{s}, \bar{z} < 1$.

The Symmetric Allocation

PROPOSITION 1. THE SYMMETRIC ALLOCATION: Given K units of capital, GDP is

$$Y = \phi(\bar{z})(S_\theta^{1-\sigma} S_\rho^\sigma)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha},$$

where

$$S_\rho \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}$$

and

$$\phi(\bar{z}) \equiv ((1 - \bar{z})^{1-\sigma} \bar{z}^\sigma)^{\frac{1}{1-\sigma}}$$

and S_θ is defined in a way analogous to S_ρ .

1. Substitution vs. Complementarity

$$S \equiv S_{\theta}^{1-\sigma} S_{\rho}^{\sigma}, \quad S_{\eta} \equiv \left(\int_0^1 A_i^{\eta} di \right)^{\frac{1}{\eta}}$$

- TFP involves both CES combinations of productivities.
 - S_{θ} is between geometric and arithmetic means
 - S_{ρ} is between geometric and minimum

⇒ Weak links crucial; importance of σ .
- Example: $\theta = 1, \rho \rightarrow -\infty, \sigma = 1/2$
 - TFP = $\bar{A} \times \min\{A_i\}$.
 - Aggregate TFP is determined by the weakest link.
- U.S. and Kenya may not be so different on average but several weak links can drag down output.

2. Linkages deliver a multiplier

$$Y = S^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}$$

- TFP is the CES average raised to the power $\frac{1}{1-\sigma}$.
- Example: Suppose $Y_t = aX_t^\sigma$ and $X_t = sY_{t-1}$.
 - Output depends on intermediate goods
 - Intermediate goods are yesterday's output.

Solving these two equations in steady state gives

$$Y^* = a^{1/1-\sigma} s^{\sigma/1-\sigma}.$$

- Analogous to the multiplier from capital accumulation.

The Competitive Equilibrium Allocation

- Standard CE with one key difference:
 - Each variety i producer is subject to a variety-specific tax τ_i
- Motivated by Banerjee-Duflo (2005), CKM (2007), Restuccia-Rogerson (2007), Hsieh-Klenow (2007)
 - Misallocation at micro level \Rightarrow Aggregate TFP.
 - Tax wedges: formal taxes, theft, expropriation, preferential credit, protection from competition, etc.
- Standard neoclassical model \Rightarrow small effects
 - Here, multiplied though intermediate goods and weak links

CE Optimization Problems

- Final Use Problem

$$\max_{\{c_i\}} \left(\int_0^1 c_i^\theta di \right)^{1/\theta} - \int_0^1 p_i c_i di$$

- Intermediate Use Problem

$$\max_{\{z_i\}} q \left(\int_0^1 z_i^\rho di \right)^{1/\rho} - \int_0^1 p_i z_i di$$

- Variety i 's Problem

$$\max_{\{X_i, K_i, H_i\}} (1 - \tau_i) p_i A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma - (r + \delta) K_i - w H_i - q X_i.$$

Definition of Equilibrium

The *competitive equilibrium with tax wedges* consists of quantities and prices $\{p_i\}, q, w, r$ such that

1. Firms and households optimize (previous slide).
2. Prices clear markets.
3. Government's budget balances: $T = \int_0^1 \tau_i p_i Y_i di$
4. Economic environment is respected.

Solving for the CE

PROPOSITION 2. THE COMPETITIVE EQUILIBRIUM, GIVEN CAPITAL: Given K , GDP in the competitive equilibrium is

$$Y = \psi(\tau) (Q_\theta^{1-\sigma} Q_\rho^\sigma)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha},$$

where

$$Q_\eta \equiv \left(\int_0^1 (A_i(1 - \tau_i))^{\frac{\eta}{1-\eta}} di \right)^{\frac{1-\eta}{\eta}},$$

and

$$\psi(\tau) \equiv \frac{1 - \sigma(1 - \tau)}{1 - \tau} \cdot \sigma^{\frac{\sigma}{1-\sigma}}$$

where $\tau \equiv T/(Y + qX)$ is an average tax rate.

Three Remarks

1. The intermediate goods multiplier plays same role.
2. Tax wedges work through aggregate TFP
 - As in CKM, RR, HK
 - Now they get multiplied by IG multiplier as well.
3. Change in curvature parameter in CES... (next slide)

Strengthen weak links, Favor superstars

$$Q_\eta \equiv \left(\int_0^1 (A_i(1 - \tau_i))^{\frac{\eta}{1-\eta}} di \right)^{\frac{1-\eta}{\eta}}$$

- Curvature parameter is $\frac{\eta}{1-\eta}$ rather than η
 - $\rho \in [0, -\infty)$ implies $\frac{\rho}{1-\rho} \in [0, -1)$
 - $\theta \in [0, 1)$ implies $\frac{\theta}{1-\theta} \in [0, \infty)$
 - A *higher* power mean
⇒ Strengthen weak links, favor superstars.
- Example: $\theta = 1, \rho \rightarrow -\infty, \sigma = 1/2$
 - TFP = $\max\{A_i\} \times \bar{A}$.
 - Aggregate TFP is determined by the superstar.
- Even with Leontief, other margins of substitution:
 - Resources substitute for low A_i .

The Steady State

PROPOSITION 3. THE COMPETITIVE EQUILIBRIUM IN STEADY STATE: Let $y \equiv Y/\bar{L}$. GDP per worker in SS is

$$y^* = \psi(\tau) (Q_\theta^{1-\sigma} Q_\rho^\sigma)^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}} \left(\frac{\alpha(1-\sigma)}{\lambda + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{h}.$$

- The long-run multiplier is $\frac{1}{1-\sigma} \frac{1}{1-\alpha} = \frac{1}{1-\beta}$
- Suppose we compare 2 economies with $Q^{rich} = 2 \times Q^{poor}$
- Income ratios
 - Neoclassical ($\sigma = 0$): $2^{3/2} \approx 2.8$
 - Here ($\sigma = 1/2$): $2^{2 \times 3/2} = 2^3 = 8$.

Symmetric Taxes

PROPOSITION 4. SYMMETRIC TAX WEDGES: Suppose $\tau_i = \bar{\tau}$. Let $z \equiv \frac{qX}{Y+qX}$. Then $z^* = \sigma(1 - \bar{\tau})$, and

$$Y = (1 - z^*)z^{*\frac{\sigma}{1-\sigma}} \left(\tilde{Q}_\theta^{1-\sigma} \tilde{Q}_\rho^\sigma \right)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha},$$

Also, in steady state

$$y^* = (1 - \sigma(1 - \bar{\tau})) (1 - \bar{\tau})^{\frac{1}{1-\sigma} \frac{1}{1-\alpha} - 1} \left(\tilde{Q}_\theta^{1-\sigma} \tilde{Q}_\rho^\sigma \right)^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}} \bar{h},$$

- GDP is maximized at $\bar{\tau} = 0$ (i.e. $z^* = \sigma$).
- Why does a symmetric tax distort?
Diamond-Mirrlees/Chamley/Judd.
- Monopoly distortions would also be multiplied.

Random Productivity and Taxes

PROPOSITION 5: Let $a_i \equiv \log A_i$ and $\omega_i \equiv \log(1 - \tau_i)$ be jointly normally distributed so that $a_i \sim N(\mu_a, \nu_a^2)$ and $\omega_i \sim N(\mu_\omega, \nu_\omega^2)$ and $Cov(\omega_i, a_i) = \nu_{a\omega}$. Finally, let $\nu^2 \equiv \nu_a^2 + \nu_\omega^2 + 2\nu_{a\omega}$. Then

$$\log y^* = \underbrace{\log \left(\frac{1 - \sigma(1 - \tau)}{1 - \tau} \right)}_A + \frac{1}{1 - \sigma} \frac{1}{1 - \alpha} \left(\underbrace{(1 - \sigma) \log Q_\theta + \sigma \log Q_\rho}_B \right)$$

where

$$A = \log \left(1 - \sigma \exp \left[\mu_\omega + \frac{1}{2} \cdot \frac{1 + \rho}{1 - \rho} \nu_\omega^2 + \frac{\rho}{1 - \rho} \nu_{a\omega} \right] \right) - \left(\mu_\omega + \frac{1}{2} \cdot \frac{1 + \theta}{1 - \theta} \nu_\omega^2 + \frac{\theta}{1 - \theta} \nu_{a\omega} \right)$$

and

$$B = \mu_a + \mu_\omega + \frac{1}{2} \cdot \left((1 - \sigma) \frac{\theta}{1 - \theta} + \sigma \frac{\rho}{1 - \rho} \right) \nu^2$$

Moreover, $\frac{\partial \log y^*}{\partial \nu_\omega^2} < 0$.

III. Development Accounting

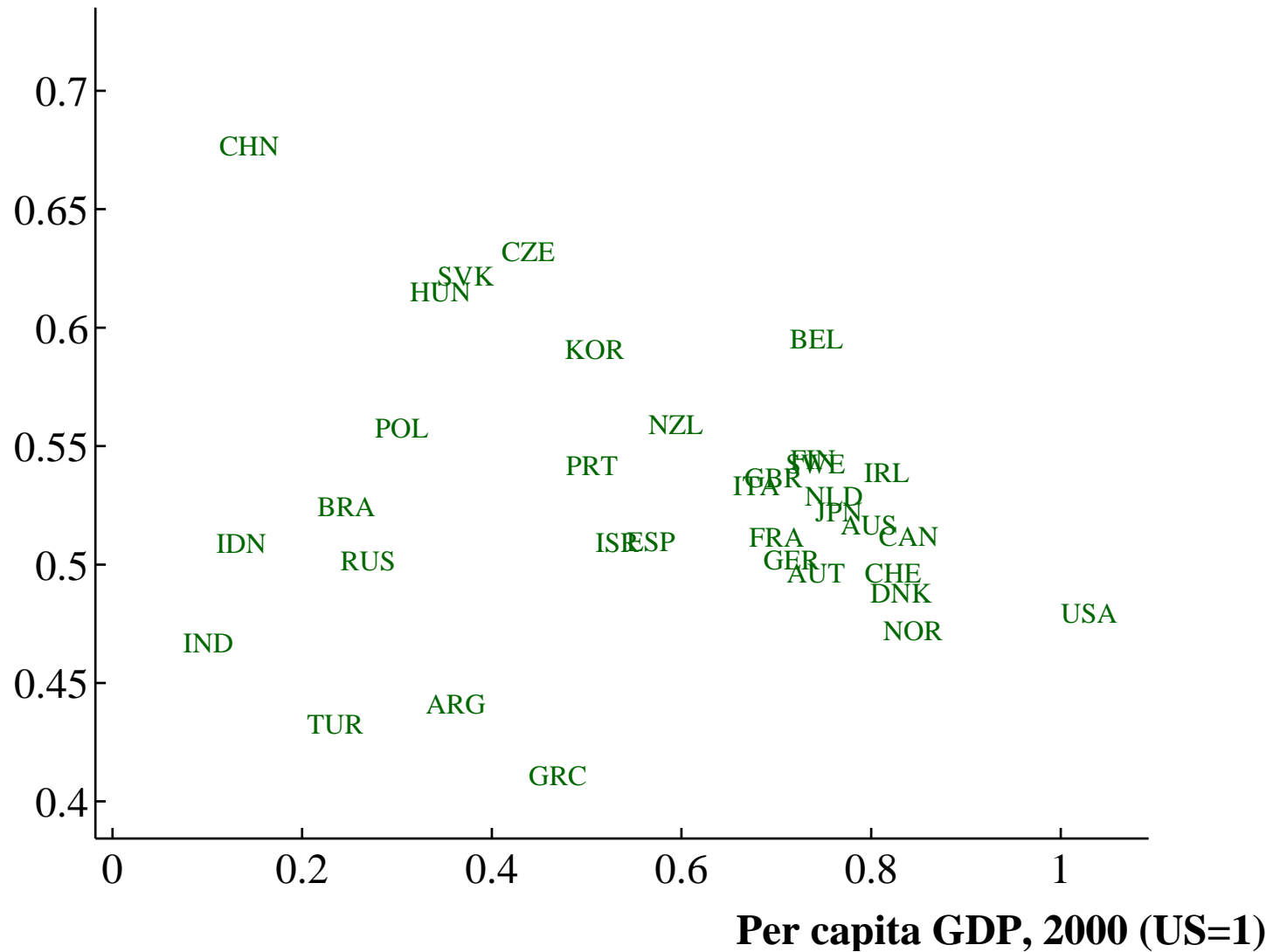
Intermediate Goods Share: σ

- Basu (1995) uses $\sigma = 1/2$ based on Jorgenson, Gollop, and Fraumeni (1987) U.S. average for 1947–1979.
- Chenery, Robinson, and Syrquin (1986) suggest that share rises with development
 - But Korea, Taiwan, and Japan in 1970s are all *higher* than this U.S. number, at 61% to 80%
- OECD I-O database at 1-digit level has
 - $\sigma \approx 46\%$ for U.S., Japan, India
 - $\sigma = 64\%$ for China
 - Across 21 countries: mean = 52.4%, stdev = 6%.

$\Rightarrow \sigma = 1/2$ seems quite reasonable

The Intermediate Goods Share

Intermediate Share, σ



Using Factor Shares to Measure Distortions?

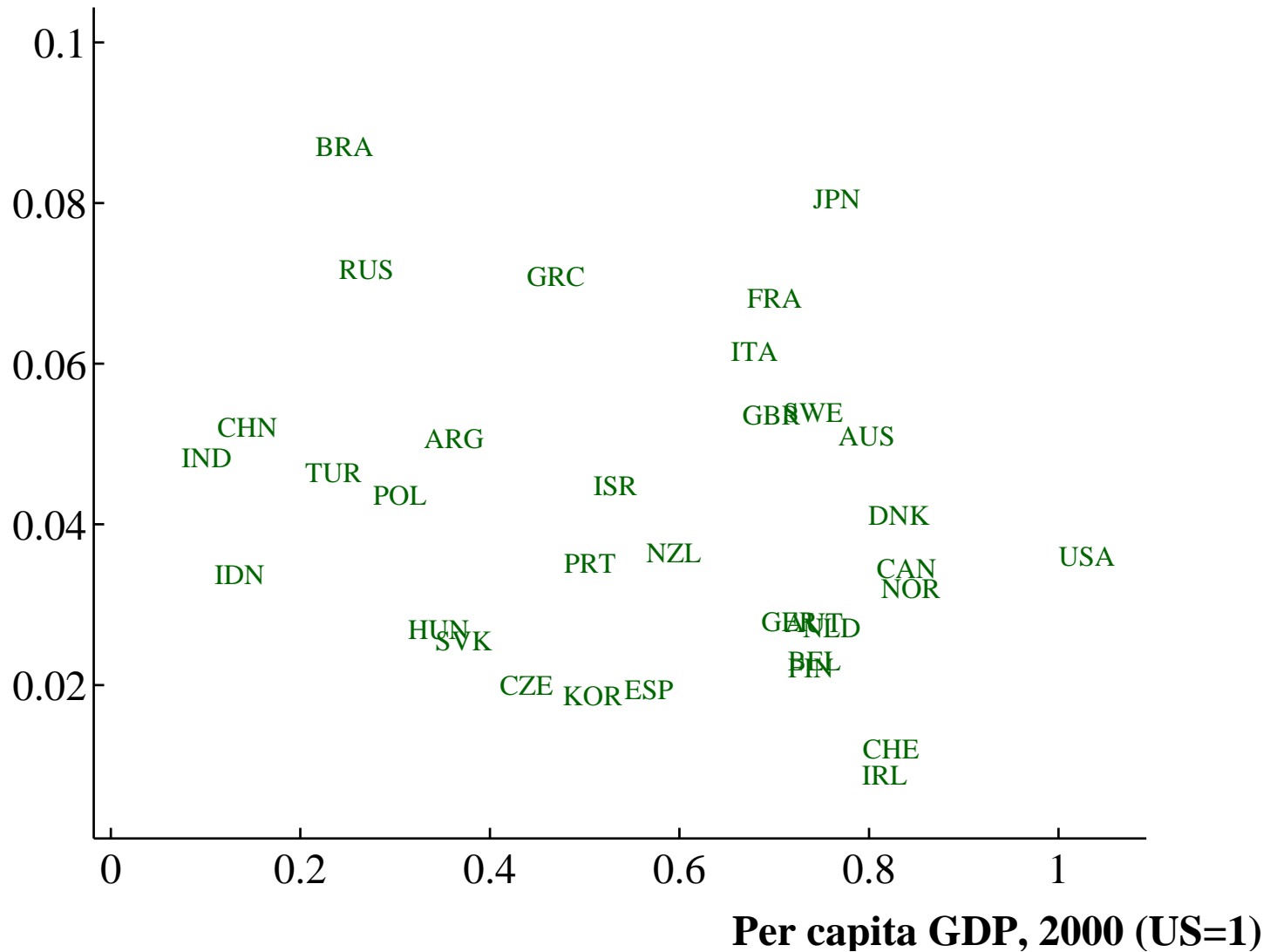
- Following the idea of Hsieh-Klenow (2007), perhaps variation in the Intermediate Share \Rightarrow distortions.
- Example: Sales tax \rightarrow FOC $\rightarrow (1 - \tau)\alpha Y/K = r$
- Similarly, for a single intermediate good (and IG taxes as well):

$$\frac{pX}{Y} = \frac{\sigma(1 - \tau_Y)}{1 + \tau_X}$$

- Problems:
 - Need to know σ_i
 - Aggregation misses firm-level distortions
 - *Measured* taxes are small.
- Conceivable that high sales taxes and large subsidies to IntGoods in China and Hungary coexist.

Effective Tax Rate from IntGoods FOCS

Effective Tax Rate



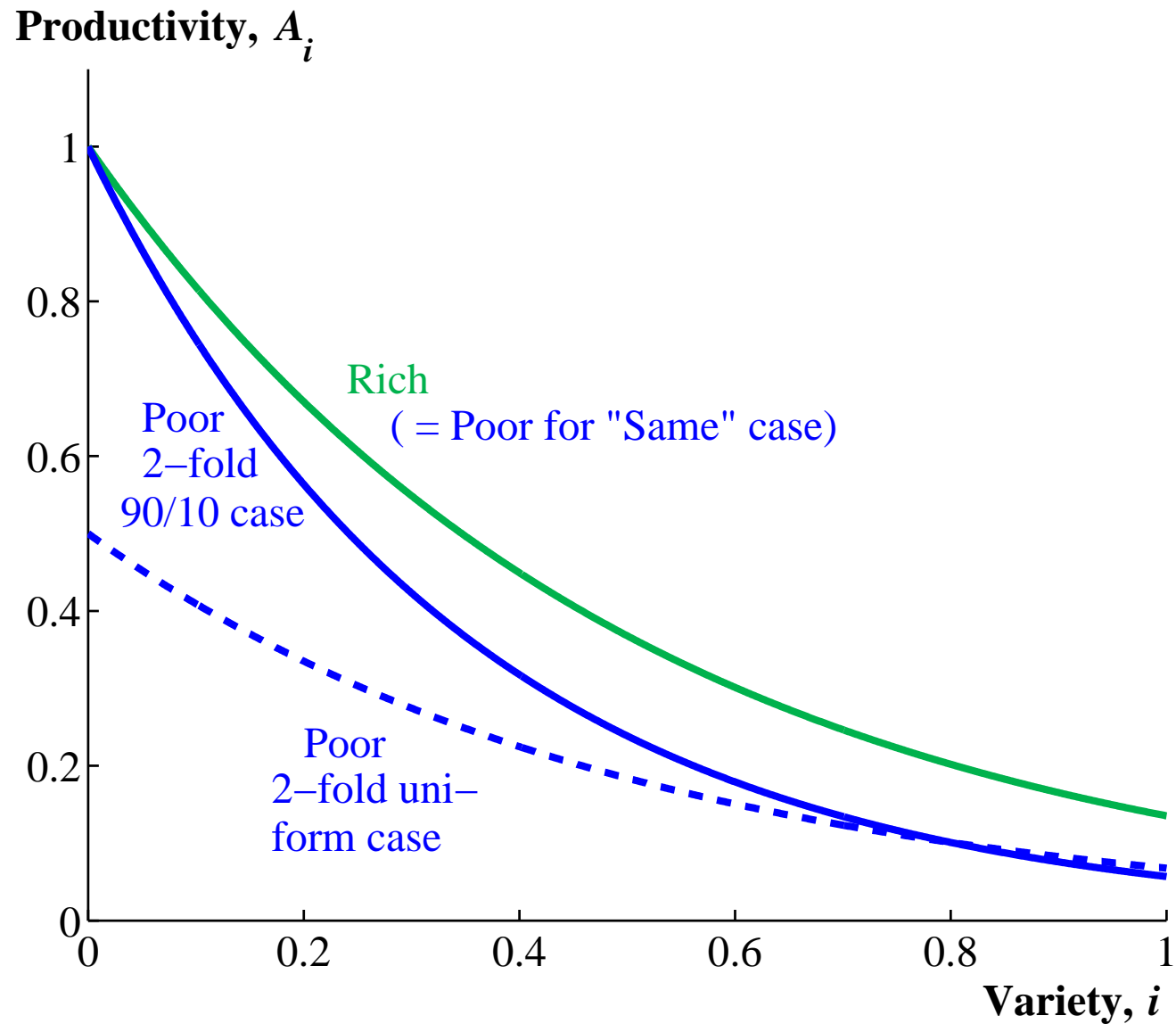
Calibrating TFP, Taxes, and Complementarity

- Least confident in calibrating: TFP, Tax Wedges, Complementarity
 - Approach: Illustrate multipliers with conservative calibration.
- Compare two countries, “rich” vs. “poor”
 1. TFP and taxes are deterministic functions of i
 2. Log-normal example tied closely to Hsieh-Klenow.

Calibrating TFP Differences

- Hsieh-Klenow: 90/10 Ratios of plant-level TFP within 4 digit industries
 - US=9, China=11, India=27
 - Not exactly what we want...
 - Baseline “rich” has a 90/10 ratio of 5
- Three scenarios:
 - “Same”: Rich and poor identical
 - “2-fold 90/10”: Identical at 100th percentile, 90/10 twice as large in poor.
 - “2-fold uniform”: Poor is 1/2 as productive in each variety.

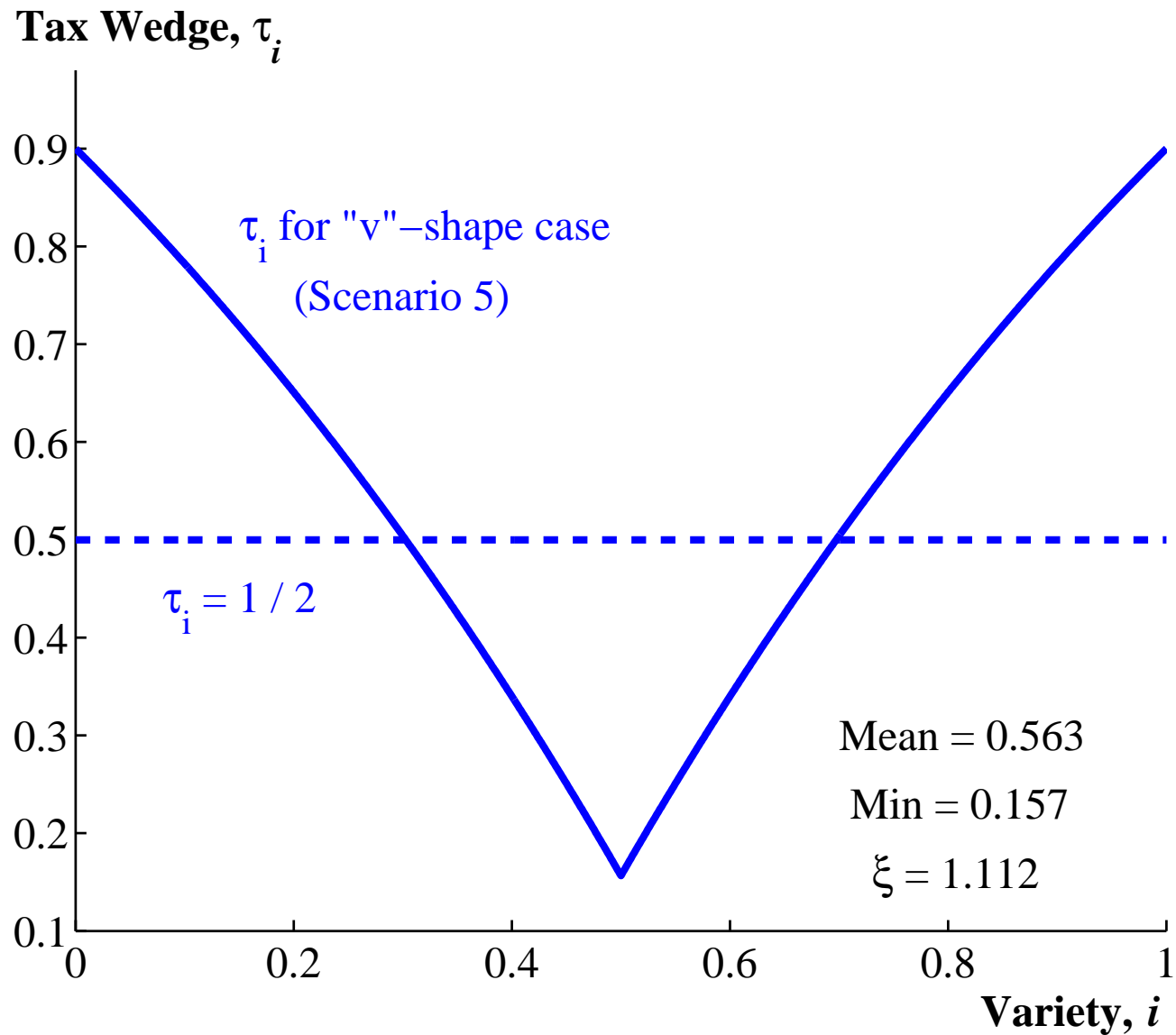
Assumed TFP: Three Scenarios



Tax Wedges

- Initially: Two interesting examples. Later: Hsieh-Klenow.
- Rich country has no distortions: $\tau_i^{rich} = 0$.
- Poor country, Two alternatives:
 - Symmetric case: $\tau_i^{poor} = 1/2$.
 - “v”-shaped wedges: Distorts both weak links and superstars.

Assumed Tax Wedges in Poor: Two Cases



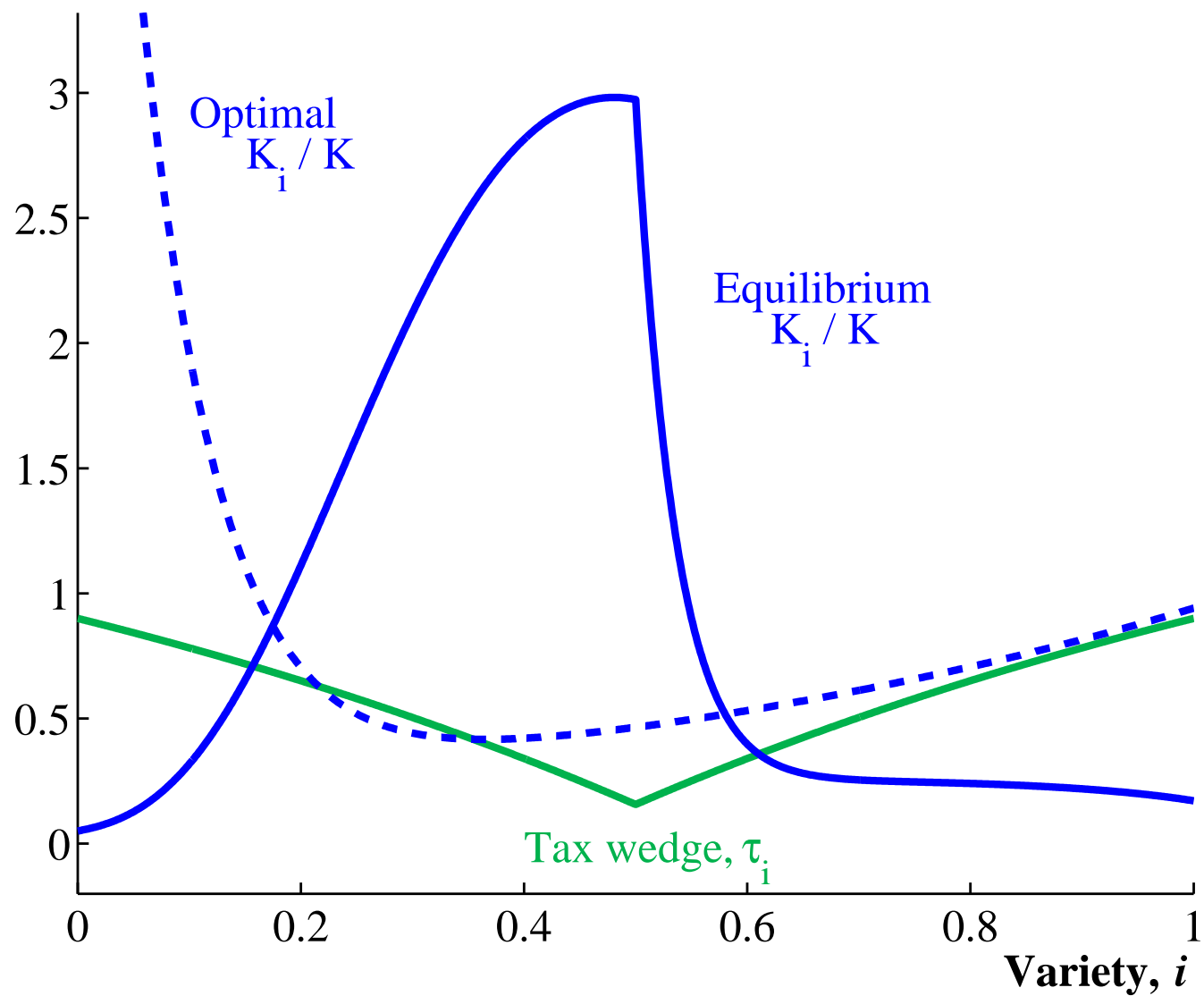
Parameter Choices

Parameter	Value	Comment
α	1/3	Conventional value for capital share
\bar{h}^r / \bar{h}^p	2	Standard contribution from education
θ	1/1.2	Consistent with 20% markups
ρ	-1	Elasticity of substitution is 1/2
\bar{A}^r	1	Normalization
\bar{A}^p	{1, 1/2}	Illustrative purposes
γ^r	2	Gives a 90/10 ratio of 4.96
γ^p	{2, 2.87}	Doubles the 90/10 ratio
$\bar{\tau}_0$	0.9	Maximum tax rate
ξ	...	To match capital-output ratio factor of 3

Output per Worker Ratios: “Rich” vs. “Poor”

Scenario	TFP Case	Tax Case	No Intermediate Goods $\sigma = 0$	Baseline Case $\sigma = 1/2$	<i>Multi- plicative Factor</i>
1.	Same	$\tau_i = 1/2$	2.8	5.3	1.9
2.	2× @ 90/10	$\tau_i = 1/2$	3.1	12.9	4.1
3.	2× uniform	$\tau_i = 1/2$	8.0	42.7	5.3
4.	Same	“v”-shape	6.3	19.6	3.1
5.	2× @ 90/10	“v”-shape	7.9	53.2	6.8
6.	2× uniform	“v”-shape	17.8	156.5	8.8

The Allocation in Scenario 5



Output per Worker Ratios: Robustness

Scenario	No Inter-	Robustness Results			
	mediates $\sigma = 0$	Cobb- Douglas	$\theta = 0.9$ (EofS=10)	$\theta = 1/2$ (EofS=2)	“Leontief” $\rho = -100$
<i>Ratio of output per worker:</i>					
1.	2.8	5.3	5.3	5.3	5.3
2.	3.1	19.6	12.3	17.5	14.4
3.	8.0	42.7	42.7	42.7	42.7
4.	6.3	8.8	24.2	11.9	22.4
5.	7.9	32.2	62.3	40.7	72.9
6.	17.8	70.2	193.3	95.1	178.9

Output per Worker Ratios: Robustness (Multipliers)

Scenario	No Inter- mediates $\sigma = 0$	Robustness Results			
	Cobb- Douglas	$\theta = 0.9$ (EofS=10)	$\theta = 1/2$ (EofS=2)	“Leontief” $\rho = -100$	
<i>Multiplicative factor relative to $\sigma = 0$:</i>					
1.	1.0	1.9	1.9	1.9	1.9
2.	1.0	6.2	3.9	5.6	4.6
3.	1.0	5.3	5.3	5.3	5.3
4.	1.0	1.4	3.8	1.9	3.5
5.	1.0	4.1	7.9	5.2	9.2
6.	1.0	3.9	10.8	5.3	10.0

A Calibration based on Hsieh-Klenow (2007)

- Does not map particularly well into this framework
 - Within industry only, not distortions between as well
 - Value-added instead of gross output
 - Distortion measure also includes distortions between factors
- Still, perhaps useful in light of limited measures.
- Focus on China and India
 - China: 8.9 times poorer than U.S.
 - India: 13.4 times poorer than U.S.

Output per Worker Ratios: Hsieh-Klenow

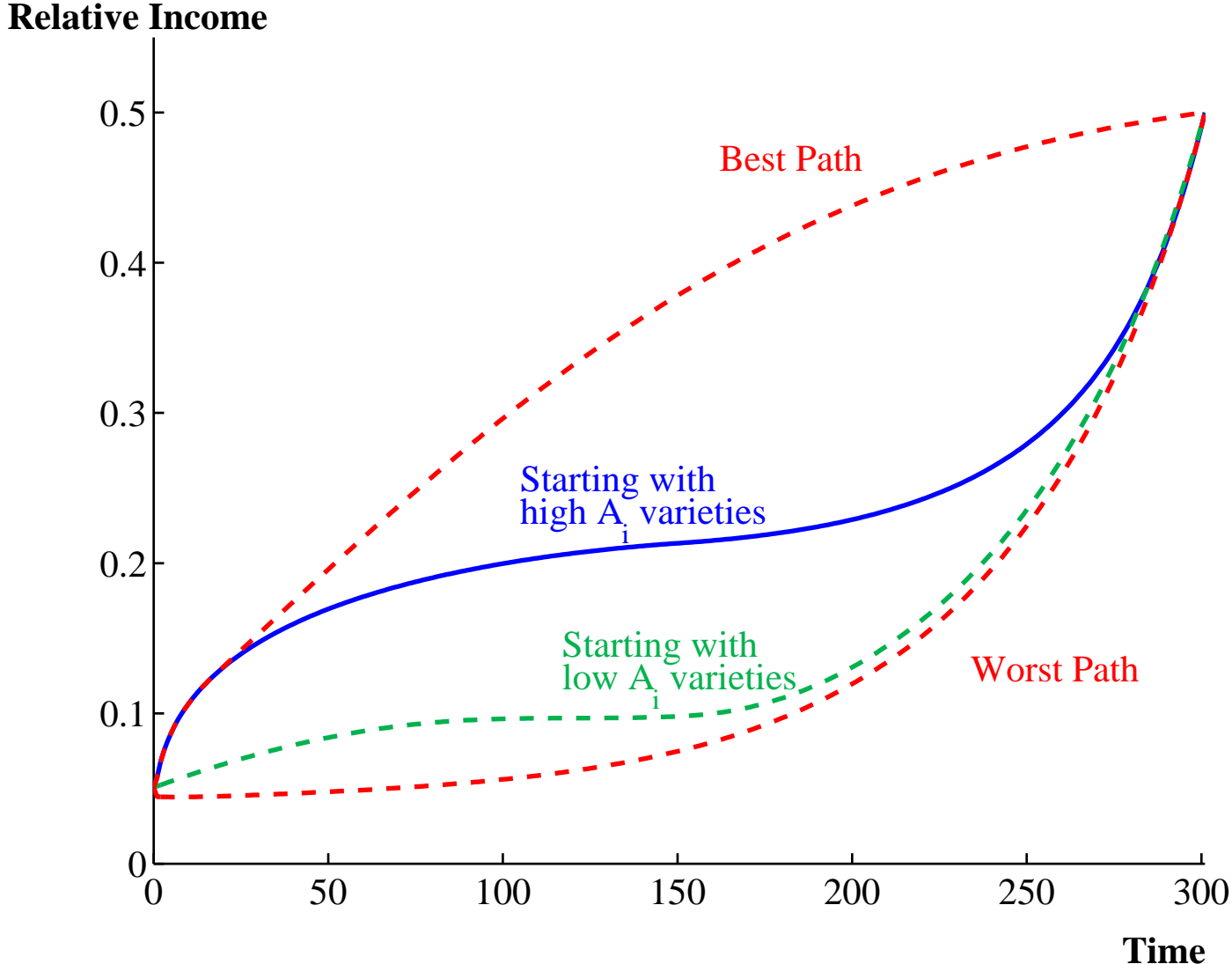
Scenario	Description	“Ave- age” TFP	No Inter- diates $\sigma = 0$	Base Case $\sigma = 1/2$	<i>Multi- plicative</i> <i>Factor</i>
7.	Baseline	0.498	3.9	14.8	3.8
8.	Identical TFPs	1.000	3.1	3.4	1.1
9.	$\nu_a^{rich} = \nu_a^{poor} = 1.07$	0.750	4.7	8.0	1.7
10.	$\nu_a^{rich} = \nu_a^{poor} = .5$	0.750	3.4	6.5	1.9
11.	$\nu_a^{rich} = .5, \nu_a^{poor} = .75$	0.559	3.7	11.9	3.2
12.	11, but $\nu_{aw} = 0$	0.559	3.0	10.5	3.5

Baseline has $\theta = 2/3$, $\rho = -1$, $\bar{h} = 1.52$. See Table 4 in paper.

IV. Making Development Hard

- A large multiplier may make the development problem easy to solve.
- However, complementarity can dampen this effect
 - If there are numerous weak links, reforms to one or two can have small effects.
- Example: Consider **Scenario 4** above
 - “V”-shaped tax wedges and a 2-fold difference in h
 - Otherwise rich and poor countries are identical
 - Sequence of reforms: eliminate one tax wedge at a time (cumulative, approximate continuum w/ 300 grid points)

Fig 5. Cumulative Effect of Reforms



Multinationals and Trade

- Some reforms may affect many sectors at once
- Trade reforms are an example
 - Import intermediate goods at weak links
 - Special economic zones
- If poorest countries are closed, the income calculations above provide a useful benchmark
 - But reform paths could be very different...
- Domestic weak links can still be crucial
 - Contract enforcement, knowledge, property rights, energy, transportation, distribution, etc.

Conclusions

- Intermediate goods and Complementarity provide multipliers
 - Intermediate goods: large effect, relatively easily calibrated.
 - Complementarity: Hard to calibrate, offset by substitution?
- Directions for further research
 - What about a much richer input-output structure?
 - Redo Hsieh and Klenow (2008) with intermediate goods
 - Measuring weak links and misallocation

N Sector Input-Output Model

$$Y_i = A_i (K_i^{\alpha_i} H_i^{1-\alpha_i})^{1-\sigma_i} x_{i1}^{\sigma_{i1}} x_{i2}^{\sigma_{i2}} \cdot \dots \cdot x_{iN}^{\sigma_{iN}}, \quad A_i = A \epsilon_i$$

$$C_j + \sum_{i=1}^N x_{ij} = Y_j$$

$$\sum_{i=1}^N K_i \leq K, \quad \sum_{i=1}^N H_i \leq H$$

$$Y = C_1^{\beta_1} C_2^{\beta_2} \cdot \dots \cdot C_N^{\beta_N}, \quad \sum_{i=1}^N \beta_i = 1$$

Allocating Resources and Results

Definition 1 *The symmetric allocation has $K_i = \bar{K} \equiv K/N$, $H_i = \bar{H} \equiv H/N$, and $x_{ij} = \bar{x}Y_j$.*

Proposition 0.1 *Let $\beta \equiv$ vector of β_i and $B \equiv$ matrix of σ_{ij} . Under the symmetric allocation,*

$$Y = A^\phi \bar{K}^{\tilde{\alpha}} \bar{H}^{1-\tilde{\alpha}} \epsilon \quad (1)$$

where

$$\phi \equiv \beta'(I - B)^{-1}\mathbf{1},$$

$$\tilde{\alpha} \equiv \sum_i \sum_j \beta_i l_{ij} (1 - \sigma_j) \alpha_j,$$

$$l_{ij} \equiv \text{elements of Leontief inverse } (I - B)^{-1}.$$

Some Corollaries

Corollary 1 *Under the symmetric allocation,*

$$\frac{\partial \log Y}{\partial \log A} = \beta'(I - B)^{-1}\mathbf{1}. \quad (2)$$

Corollary 2 *Suppose $\sum_{j=1}^N \sigma_{ij} = \bar{\sigma}$. Then under the symmetric allocation,*

$$\frac{\partial \log Y}{\partial \log A} = \frac{1}{1 - \bar{\sigma}}. \quad (3)$$

I-O Multipliers for 1-Digit Data

Country	Input-Output Multiplier $\beta'(I - B)^{-1}\mathbf{1}$	“As-if” Intermediate Share	Actual Intermediate Share
United States	1.82	0.449	0.462
Japan	1.71	0.415	0.465
China	2.60	0.615	0.641
India	1.66	0.396	0.467

Source: OECD Input-Output Database.