Bus Ad 239B–Spring 2003
Solutions to Problem Set 13

1. In the discrete setting, this is obvious. For each k,

$$\begin{aligned} & E\left(\left(\hat{X}\left(\frac{k+1}{n}\right)\right|\mathcal{F}_{k/n}\right) \\ = \;& \hat{X}\left(\frac{k}{n}\right) + \left(1\times\frac{\rho}{n} + 0\times\left(1-\frac{\rho}{n}\right)\right) - \rho\times\frac{1}{n} \\ = \;& \hat{X}\left(\frac{k}{n}\right) \end{aligned}$$

   In the continuous setting, fix $s < t$. Then, using the formula for the expectation of a Poisson process,

$$E(\hat{X}(t)|\hat{X}(s)) = \hat{X}(s) + \rho(t-s) - \rho(t-s) = \hat{X}(s)$$

   so $\hat{X}$ is a martingale with respect to the filtration it generates.

2. (a) Since X is adapted with respect to $\mathcal{F}_W$, So is $\hat{X}$. If $\hat{X}$ is a martingale with respect to $\mathcal{F}^W$, then by the Martingale Representation Theorem, $\hat{X}$ is an Itô Process, and hence is continuous almost surely, which is a contradiction. Therefore, $\hat{X}$ is *not* a martingale with respect to $\mathcal{F}^W$.

   (b) The problem was intended to say $e^{\hat{X}+W}$, but the answer is the same in both cases: *No.* The fact that $\hat{X}$ is adapted but not a martingale with respect to $\mathcal{F}^W$, while it is a martingale with respect to $\mathcal{F}^X$, tells us that $\mathcal{F}_t^W$ contains more information about the future of $\hat{X}$ than $\mathcal{F}_t^X$ does. In particular, the distribution of jumps after time t is independent of $\mathcal{F}_t^X$ and stationary in t. However, if $\hat{X}$ is adapted to $\mathcal{F}^W$, then since $\mathcal{F}^W$ is a continuous filtration, $\mathcal{F}^W$ signals that a jump is highly likely to occur just before it occurs. Accordingly, traders will sell short the Money-Market account in order to buy the stock when the information inf $\mathcal{F}^W$ signals that a jump is imminent, driving up the price of the stock to the level it would attain once the jump is confirmed.