Bus Ad 239B–Spring 2002
Problem Set 7
Due Thursday March 13

Assume a 3-dimensional security price process $\bar{S}$ is given by

$$\bar{S}_n(\omega, t) = e^{\bar{Z}_n(\omega,t)} \quad (n = 0, 1, 2)$$

where

$$\bar{Z}_n(\omega, t) = \bar{Z}_n(\omega, 0) + \int_0^t \bar{\mu}_n - \frac{\bar{\sigma}_n \bar{\sigma}_n^T}{2}\, dt + \int_0^t \bar{\sigma}_n \, dW$$

and $\bar{\mu}$ is a 3×1 constant process, $\bar{\sigma}$ is a 3×2 constant process, $\bar{\sigma}_n$ is the n^{th} row of $\bar{\sigma}$, W is a 2-dimensional standard Wiener process, and rank $\bar{\sigma} = 2$.

1. Show that, for almost all $\bar{\sigma}$, there is a unique 1×3 process $\bar{b}_0 \in \mathcal{L}(S)$ such that $\int_0^t \bar{b}_0 \, d\bar{S}$ is deterministic and $\bar{b}_0 \bar{S} = 1$ for all t. In other words, find conditions on $\bar{\sigma}$ which ensure that $\bar{b}$ exists and is unique, then argue that the set of matrices $\bar{\sigma}$ on which these conditions fail is a set of Lebesgue measure zero in $\mathbf{R}^{3\times 2}$. For the remaining parts of the problem, assume that this condition on $\bar{\sigma}$ is satisfied.

2. Find necessary and sufficient conditions on $\bar{\mu}$ and $\bar{\sigma}$ such that the process $\bar{b}_0$ found in part (1) is self-financing.

3. Now suppose that $\bar{\sigma}$ and $\bar{\mu}$ don't necessarily satisfy the conditions you found in part (2). Find the stochastic differential equation for $\bar{b}$ such that $\int_0^t \bar{b} \, d\bar{S}$ is deterministic and $\bar{b}$ is self-financing.

4. Solve the stochastic differential equation you found in part (3) and determine the interest rate process.

5. Is there a state price process for $\bar{S}$. If so, find it.