

Bus Ad 239B–Spring 2003
Problem Set 6
Due Thursday March 6

1. (Carried over from Problem Set 5) Consider the random walk version of the Brownian bridge discussed in class, i.e Fix n even, T such that $nT \in \mathbf{N}$, let X be the standard binary random walk described in Lecture 1, and consider the conditioned process

$$B(\omega, t) = X(\omega, t | X(\omega, T) = 0)$$

As noted in class, if we define $\Omega' = \{\omega \in \Omega : X(\omega, T) = 0\}$ with the probability measure $P(A) = \frac{|A|}{|\Omega'|}$, then we can think of B as the restriction of X to Ω' . Compute $\sigma^2(\omega, k, n)$, the variance of $B\left(\omega, \frac{k+1}{n}\right) - B\left(\omega, \frac{k}{n}\right)$. Now suppose $n \rightarrow \infty$ and $\frac{k_n}{n} \rightarrow t \in [0, T)$. Show that

$$n\sigma^2(\omega, k_n, n) \rightarrow 1$$

(you will need to decide what sense of convergence to use). What happens if $k_n/n \rightarrow T$? Use these facts to derive (you can be informal) the stochastic differential equation satisfied by the Brownian bridge

$$W(\omega, t | W(\omega, T) = 0)$$

where W is a standard Wiener process.