

Bus Ad 239B–Spring 2003
Problem Set 4
Due Thursday February 20

In some of the following problems, the note on page 59 of Nielsen regarding functions of time will be helpful.

1. Use Itô's Lemma to compute $\int_0^T (W(\omega, t))^n dW(\omega, t)$ and $\int_0^T W(\omega, t) dt$, where W is a standard 1-dimensional Wiener process.
2. Let $W = (W_1, W_2)$ be a standard 2-dimensional Wiener process. Use Itô's Lemma to express the following as Itô processes

(a)

$$Z(\omega, t) = e^{(W_1(\omega, t))^2}$$

(b)

$$Z(\omega, t) = (W_1(\omega, t) + W_2(\omega, t))^2$$

(c)

$$Z(\omega, t) = e^{W_1(\omega, t) + 2W_2(\omega, t)}$$

(d)

$$Z(\omega, t) = e^{t^2 + W_1(\omega, t)}$$

(e)

$$Z(\omega, t) = te^{W_1(\omega, t)}$$

3. Let

$$Z(\omega, t) = \max\{W(\omega, s) : s \in [0, t]\}$$

where W is a standard 1-dimensional Wiener process. Is Z an Itô process?