Bus Ad 239B–Spring 2003
Problem Set 3
Due Thursday February 13

Itô's Lemma for the Random Walk: Let X_n be the 1-dimensional random walk of Lecture 1, and f a C^2 function. Show that if n is large,

$$f(X_n(\omega,T)) \simeq f(X_n(\omega,0)) + \int_0^T f'(\hat{X}_n(\omega,t))dX_n(\omega,t) + \frac{1}{2}\int_0^T f''(X_n(\omega,t))dt$$

Hint: Write

$$f(X_n(\omega,T)) = f(X_n(\omega,0)) + \sum_{k=1}^{nT}\left[f\left(X_n\left(\omega,\frac{k+1}{n}\right)\right) - f\left(X_n\left(\omega,\frac{k}{n}\right)\right)\right]$$

Use Taylor's formula. Recall that for the random walk, the integral

$$\int_0^T f'(\hat{X}_n(\omega,t))dX_n(\omega,t)$$

is defined as a Stieltjes integral.