Bus Ad 239B–Spring 2003
Problem Set 2
Due Thursday February 6

1. Consider a 2-dimensional generalized Brownian motion $B(\omega, t)$ with $B(\omega, 0) = 0$, mean zero, and covariance matrix
$$t \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$
Construct a standard 2-dimensional Brownian motion $\hat{B}$ and a 2×2 matrix σ such that
$$B(\omega, t) = \sigma \hat{B}(\omega, t)$$
Verify that
$$\sigma \sigma^T = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

2. Let B be a standard 1-dimensional Brownian motion. Show that the stochastic process
$$S(\omega, t) = e^{\alpha t + \beta B(\omega, t)}$$
is a martingale with respect to the filtration it generates if and only if $\alpha = -\frac{\beta^2}{2}$.

3. Let X_n and $\hat{X}_n$ be the random walks defined in the first lecture. Show that
$$\int_0^T \hat{X}_n(\omega, t) dX_n(\omega, t) = \frac{1}{2}(X_n(\omega, T)^2 - T)$$
For each ω, the integral on the left hand side is a Stieltjes integral taken with respect to the path $X_n(\omega, \cdot)$. Compute
$$\int_0^T X_n(\omega, t) dX_n(\omega, t)$$
and explain why you get a different answer.